

Minsk, May 2020

**Form Factor for a System of the Two Fermions an Equal  
Masses in the Relativistic Quasipotential Approach**

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## Plan of Talk

- 1. Introduction
- 2. Equation for the wave function of a System of the Two Fermions
- 3. Form Factor for a System of the Two Fermions
- 4. Root-mean-square radius and form factor for Coulomb interaction
- 5. Conclusions

## 1. Introduction

To describe the behavior of the form factors the different pole vector-dominance models (VDM) are often used [J. G. Korner, M. Kuroda, *Phys. Rev. D* **16**, 2165 (1977); S. Dubnicka, *Nuovo Cimento A* **103**, 1417 (1991)]. However the models VDM fail in description experimentally of the observed for large importances of the momentum transfer of the system  $-t = Q^2$  the quick decrease of electromagnetic form factor at time-like region according to the law of dipole  $\sim t^{-2}$ .

The using of three-dimensional relativistic quasipotential (RQP) equation of Logunov-Tavkhelidze [A. A. Logunov, A. N. Tavkhelidze, *Nuovo Cim.* **29**, 380 (1963)] for description of the form factors of composite systems was executed in [V. A. Matveev, R. M. Muradyan, A. N. Tavkhelidze, Preprint No. E2-3498, JINR (Dubna, 1967); No. P2-3900, JINR (Dubna, 1968); V. R. Garsevanishvili, A. N. Kvinikhidze, V. A. Matveev, A. N. Tavkhelidze, and R. N. Faustov, *Theor. Math. Phys.* **23**, 533 (1975); A. L. Khelashvili, Preprint No. P2-8750, JINR (Dubna, 1976); R. N. Faustov, *Theor. Math. Phys.* **3**, 478 (1970); *Ann. Phys.* **78**, 176 (1973)]. However, use of the equation Logunov-Tavkhelidze for wave function in the momentum representation has not allowed to research the behavior of form factor in broad interval of importances of momentum transfer of relativistic two-particle bound system.

The other model in which the contribution of small distances in the proton form factor take into account was considered in [N. B. Skachkov, I. L. Solovtsov, Preprint No. E2-11727, JINR (Dubna, 1978); *Yad. Fiz.* **30**, 1079 (1979); *Teor. Mat. Fiz.* **43**, 330 (1980)]. This model is based on invariant description of the structure of the particles in relativistic configurational space that was carried in [V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, *Nuovo Cimento A* **55**, 233 (1968)] in the case of interaction between two relativistic spinless particles with equal masses  $m$  in which the Compton wavelength of particle plays role of natural scale.

In the **RQP** approach for a bound system of two relativistic spinless particles of arbitrary masses developed in [V. G. Kadyshevsky, M. D. Mateev, and R. M. Mir-Kasimov, *Yad. Fiz.* **11**, 692 (1970); V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, *Fiz. Elem. Chastits At. Yadra* **2**, 635 (1972)], new covariant expressions of the components of elastic form factor for the cases of a scalar and vector currents as functions of the invariant variable  $\Delta_{\mathcal{P},\mathcal{Q}}^2$ , which there is the square of the momentum-transfer vector in the Lobachevsky space, have been found in [Yu. D. Chernichenko, *Phys. At. Nucl.* **77**, 229 (2014); **78**, 201 (2015)].

The aim of present study, considered as continuation in [Yu. D. Chernichenko, *Phys. At. Nucl.* **77**, 229 (2014); **78**, 201 (2015)], is to obtain the new expression for the elastic form factor of the two relativistic with equal masses fermions bound state in the case of scalar and vector currents. Consideration is conducted within the framework of RQP approach [V. G. Kadyshevsky, *Nucl. Phys. B* **6**, 125 (1968); V. G. Kadyshevsky and M. D. Mateev, *Nuovo Cimento A* **55**, 275 (1968)] by transition to the three-dimensional relativistic configurational representation in the case of two relativistic spin particles with equal masses [V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, *Nuovo Cimento A* **55**, 233 (1968)].

## 2. Equation for the wave function of a System of the Two Fermions

Within the framework of RQP approach [V. G. Kadyshevsky, Nucl. Phys. B **6**, 125 (1968); V. G. Kadyshevsky and M. D. Mateev, Nuovo Cimento A **55**, 275 (1968)] for spherically symmetric potentials equation for the wave function in the  $\mathbf{r}$ -representation [V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, Nuovo Cimento A **55**, 233 (1968)] in the case of two relativistic particles with equal masses  $m$  and spin  $1/2$  has the form [Yu. D. Chernichenko, Phys. At. Nucl. **80**, 707 (2017)] ( $\hbar = c = 1$ )

$$\frac{1}{2m}(M_{\mathcal{Q}} - \hat{H}_0)\psi_{M_{\mathcal{Q}}}(\mathbf{r}) = V(\mathbf{r})\hat{A}\left(\frac{\hat{H}_0}{2m}\right)\psi_{M_{\mathcal{Q}}}(\mathbf{r}). \quad (1)$$

Here  $M_{\mathcal{Q}}^2 = s_q = \mathcal{Q}^2 = (q_1 + q_2)^2 = \mathcal{Q}_0^2 - \mathcal{Q}^2$ , where



$q_i, i = 1, 2 - 4$ -momentas of composite particle, the operator

$$\hat{H}_0 = 2m \left[ \cosh \left( i\lambda \frac{\partial}{\partial r} \right) + \frac{i\lambda}{r} \sinh \left( i\lambda \frac{\partial}{\partial r} \right) - \frac{\lambda^2}{2r^2} \Delta_{\theta, \varphi} \exp \left( i\lambda \frac{\partial}{\partial r} \right) \right] \quad (2)$$

is the operator of free Hamiltonian while  $\Delta_{\theta, \varphi}$  is its the angular part, and  $\lambda = 1/m$  is the Compton wavelength; quasipotential  $V(\mathbf{r})$  is local in the sense of Lobachevsky geometry, the group parameter  $r$  plays the role of modulus of relativistic relative coordinate  $\mathbf{r}$  ( $\mathbf{r} = r\mathbf{n}, |\mathbf{n}| = 1$ ), and the operator  $\hat{A}$  is defined as

$$\hat{A} \left( \frac{\hat{H}_0}{2m} \right) = \frac{1}{4} \left[ a \left( \frac{\hat{H}_0}{2m} \right)^2 + b \right], \quad (3)$$

$$a = \begin{cases} 1, & \hat{O} = \gamma_5 \text{ (pseudoscalar);} \\ 1/2, & \hat{O} = \gamma_\mu \text{ (vector);} \\ -1/2, & \hat{O} = \gamma_5 \gamma_\mu \text{ (pseudovector);} \end{cases} \quad b = \begin{cases} 0, & \hat{O} = \gamma_5; \\ 1/4, & \hat{O} = \gamma_\mu; \\ 3/4, & \hat{O} = \gamma_5 \gamma_\mu. \end{cases}$$

For simplicity, we assume, in just the same way as in [Yu. D. Chernichenko, Phys. At. Nucl. **80**, 707 (2017); N. B. Skachkov and I. L. Solovtsov, Preprint No. E2-81-760, JINR (Dubna, 1981); Theor. Math. Phys. **54**, 116 (1983)], that the quasipotential has a bispinor structure of the  $I \otimes I$  form and that the vertex function also has a specific momentum-variable-independent spinor structure that is proportional to the matrix  $\hat{O}$ ; for  $\hat{O}$ , we choose the Dirac matrices  $\gamma_5, \gamma_\mu$ , and  $\gamma_5 \gamma_\mu$  ( $\mu = 0, 1, 2, 3$ ). This choice of the matrix  $\hat{O}$  has allowed us find exact solutions of the RQP equation (1) [Yu. D. Chernichenko, Phys. At. Nucl. **80**, 707 (2017)].

By using the expansion of the RQP wave function  $\psi_{M_{\mathcal{Q}}}(\mathbf{r})$

$$\psi_{M_{\mathcal{Q}}}(\mathbf{r}) = \sum_{\ell=0}^{\infty} (2\ell + 1) i^{\ell} \frac{\varphi_{\ell}(r, \chi)}{r} P_{\ell} \left( \frac{\Delta_{q,m\lambda_{\mathcal{Q}}} \cdot \mathbf{r}}{|\Delta_{q,m\lambda_{\mathcal{Q}}}| r} \right),$$

Eq. (1) transformed to the equation for the partial wave function:

$$\left( \hat{H}_{0,\ell}^{\text{rad}} - \cosh \chi \right) \varphi_{\ell}(r, \chi) = -V(r) \hat{A} \left( \hat{H}_{0,\ell}^{\text{rad}} \right) \varphi_{\ell}(r, \chi). \quad (4)$$

Here the operator  $\hat{A}$  is defined in (3), the operator

$$\hat{H}_{0,\ell}^{\text{rad}} = \cosh \left( i\lambda \frac{d}{dr} \right) + \frac{\lambda^2 \ell(\ell + 1)}{2r(r + i\lambda)} \exp \left( i\lambda \frac{d}{dr} \right) \quad (5)$$

is the radial part of free Hamiltonian operator (2), and  $\chi$  is the rapidity related with the relative 3-momentum and energy by the formulas:

$\Delta_{q,m\lambda_Q} = m \sinh \chi \mathbf{n}_{\Delta_{q,m\lambda_Q}}, |\mathbf{n}_{\Delta_{q,m\lambda_Q}}| = 1, M_Q =$   
 $2\Delta_{q,m\lambda_Q}^0, \Delta_{q,m\lambda_Q}^0 = m \cosh \chi,$  where  $\Delta_{q,m\lambda_Q}^0$  and  $\Delta_{q,m\lambda_Q}$  are,  
 respectively, the time and spatial components of the 4-vector  
 $\Lambda_{\lambda_Q}^{-1} q = \Delta_{q,m\lambda_Q}$  from the Lobachevsky space with the velocity  
 4-vector of the composite particle  $\lambda_Q = (\lambda_Q^0; \boldsymbol{\lambda}_Q) = Q/\sqrt{Q^2}$ :

$$\Lambda_{\lambda_Q}^{-1} \mathbf{q} = \Delta_{q,m\lambda_Q} = \mathbf{q}(-)m\boldsymbol{\lambda}_Q = \mathbf{q} - \boldsymbol{\lambda}_Q \left( q_0 - \frac{\mathbf{q} \cdot \boldsymbol{\lambda}_Q}{1 + \lambda_Q^0} \right), \quad (6)$$

$$(\Lambda_{\lambda_Q}^{-1} q)^0 = \Delta_{q,m\lambda_Q}^0 = q_0 \lambda_Q^0 - \mathbf{q} \cdot \boldsymbol{\lambda}_Q = \sqrt{m^2 + \Delta_{q,m\lambda_Q}^2},$$

and all 4-momenta belong to the upper sheet of the mass hyperboloid

$$\Delta_{q,m\lambda_Q}^2 = \Delta_{q,m\lambda_Q}^{02} - \Delta_{q,m\lambda_Q}^2 = m^2. \quad (7)$$

### 3. Form Factor for a System of the Two Fermions

In ref. [N. B. Skachkov, I. L. Solovtsov, Preprint No. E2-11727, JINR (Dubna, 1978); *Yad. Fiz.* **30**, 1079 (1979)], founded on refs. [V. A. Matveev, R. M. Muradyan, A. N. Tavkhelidze, Preprint No. E2-3498, JINR (Dubna, 1967); No. P2-3900, JINR (Dubna, 1968); V. R. Garsevanishvili, A. N. Kvinikhidze, V. A. Matveev, A. N. Tavkhelidze, and R. N. Faustov, *Theor. Math. Phys.* **23**, 533 (1975); A. L. Khelashvili, Preprint No. P2-8750, JINR (Dubna, 1976); R. N. Faustov, *Theor. Math. Phys.* **3**, 478 (1970); *Ann. Phys.* **78**, 176 (1973)], the form factor of two-particle system was defined as the matrix element of the local current operator between bound states with the 4-momentum  $Q$  and  $P$  through the covariant wave RQP-functions satisfying RQP-equation in the momentum representation. Then, the invariant expression in the momentum representation for the matrix element of the local current operator of the two relativistic with equal masses fermions bound state

in the case of scalar current has the form

$$\begin{aligned}
 \langle \mathcal{P} | J | \mathcal{Q} \rangle = & -\frac{z_1}{(2\pi)^3} \int d\tau_{\mathcal{P}} d\tau_{\mathcal{Q}} d^{(4)}k_1 d^{(4)}k'_1 d^{(4)}k_2 \times & (8) \\
 & \times \Gamma_{\mathcal{P}}^{\alpha\beta+}(k_1, k_2; \lambda_{\mathcal{P}}\tau_{\mathcal{P}}) \frac{S_{\gamma}^{(+)\alpha}(k_1, m) S_{\delta}^{(+)\gamma}(k'_1, m)}{(\tau_{\mathcal{P}} + i\varepsilon)(\tau_{\mathcal{Q}} - i\varepsilon)} \times \\
 & \times \Gamma_{\mathcal{Q}}^{\delta\kappa}(k'_1, k_2; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}) S_{\kappa}^{(+)\beta}(k_2, -m) \times \\
 & \times \delta^{(4)}(-\mathcal{Q} + k'_1 + k_2 - \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}) \delta^{(4)}(\mathcal{P} - k_1 - k_2 + \lambda_{\mathcal{P}}\tau_{\mathcal{P}}) + (1 \leftrightarrow 2).
 \end{aligned}$$

Here  $\Gamma_{\mathcal{Q}}^{\delta\kappa}(k'_1, k_2; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}})$  and  $\Gamma_{\mathcal{P}}^{\alpha\beta}(k_1, k_2; \lambda_{\mathcal{P}}\tau_{\mathcal{P}})$  are the vertex functions;  $\alpha, \beta, \dots, \kappa (= 0, 1, 2, 3)$  are bispinor indices;  $S^{(+)}(k_i, m) = \theta(k_{i0})(\hat{k}_i + m)\delta(k_i^2 - m^2)$  are the positive-frequency parts of the spinor Green's functions;  $\hat{k}_i = k_i^{\mu}\gamma_{\mu}$ , and  $k_i (p_i, q_i, i = 1, 2)$  are 4-momenta of the constituents with masses  $m_1 = m_2 = m$ , and all 4-momenta of the particles belong to the upper sheet of the mass hyperboloid (7).

The vertex functions for simplicity, same as in [Yu. D. Chernichenko, *Phys. At. Nucl.* **80**, 707 (2017)] at conclusion of the equation (1), have a specific momentum-variable-independent spinor structure that is proportional to the matrix  $\hat{O}$ ,

$$\Gamma_{\mathcal{Q}}^{\delta\kappa}(k'_1, k_2; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}) = \hat{O}^{\delta\kappa}\Gamma_{\mathcal{Q}}(k'_1, k_2; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}),$$

$$\Gamma_{\mathcal{P}}^{\alpha\beta}(k_1, k_2; \lambda_{\mathcal{P}}\tau_{\mathcal{P}}) = \hat{O}^{\alpha\beta}\Gamma_{\mathcal{P}}(k_1, k_2; \lambda_{\mathcal{P}}\tau_{\mathcal{P}}),$$

and for  $\hat{O}$ , we choose the Dirac matrices  $\gamma_5, \gamma_\mu$ , and  $\gamma_5\gamma_\mu$  ( $\mu = 0, 1, 2, 3$ ).

The expression in (8) corresponds diagram on fig. 1, where  $\tau_Q \neq \tau_P$  and  $\lambda_Q \neq \lambda_P$ ,  $\lambda_Q = Q/\sqrt{Q^2}$ ,  $Q^2 = (q_1 + q_2)^2 = s_q = M_Q^2$ ,  $\lambda_P = P/\sqrt{P^2}$ ,  $P^2 = (p_1 + p_2)^2 = s_p = M_P^2$ .

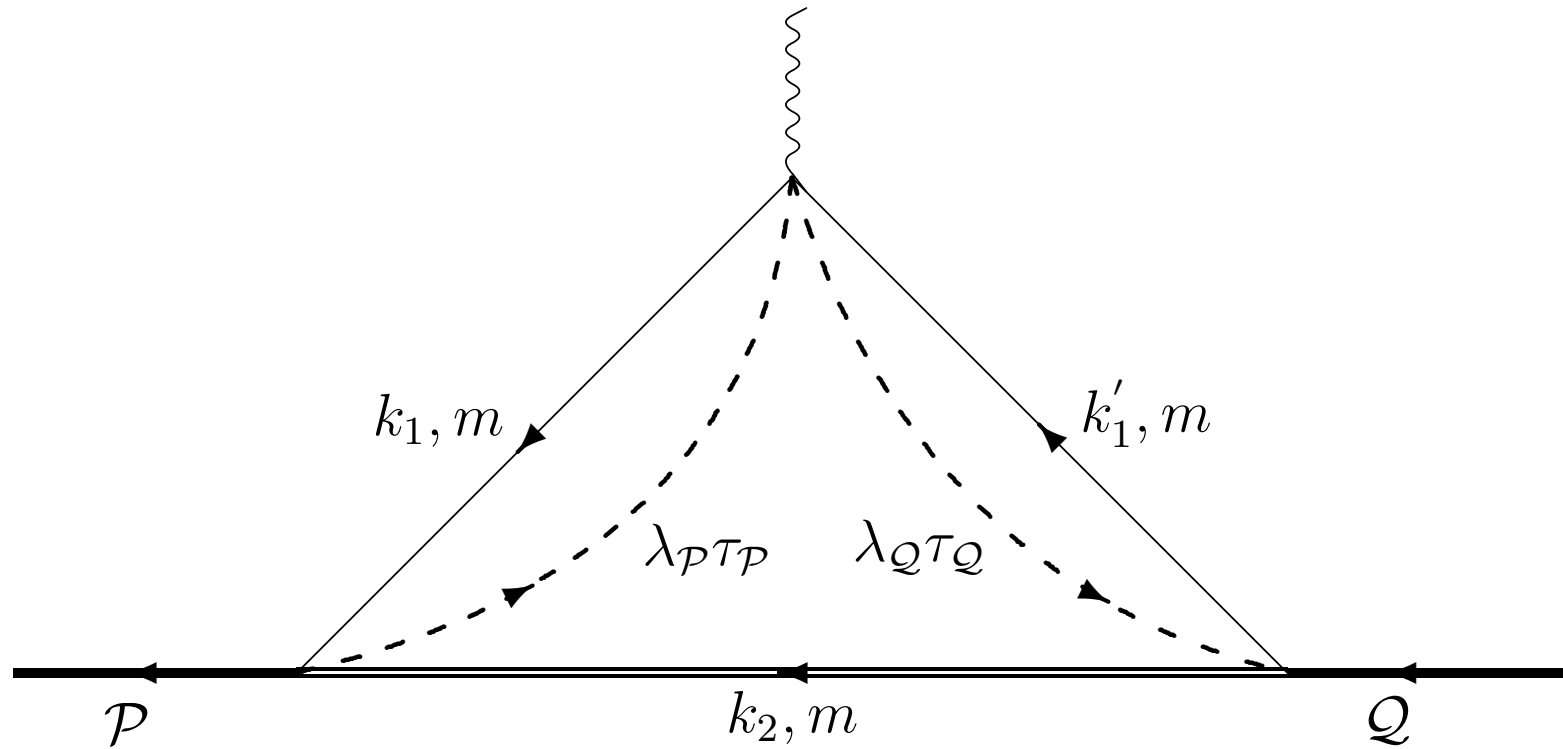


Fig. 1: The diagram of matrix element of the local current operator in the case of two relativistic spin particles with equal masses.



We execute the integration respecting of  $k'_{10}, k_{i0}, i = 1, 2$  in (8) in just the same way as we performed integration in [Yu. D. Chernichenko, Phys. At. Nucl. **77**, 229 (2014); **78**, 201 (2015)]:

$$\begin{aligned} \langle \mathcal{P} | J | \mathcal{Q} \rangle = & -\frac{z_1}{(4\pi)^3} \int \frac{d\tau_{\mathcal{P}} d\tau_{\mathcal{Q}} d\mathbf{k}_1 d\mathbf{k}'_1 d\mathbf{k}_2}{\sqrt{m^2 + \mathbf{k}_2^2}} \times \quad (9) \\ & \times \frac{\Gamma_{M_{\mathcal{P}}}^*(\mathcal{P}k_2) \text{Tr}[\hat{O}^+(\hat{k}_1 + m)(\hat{k}'_1 + m)\hat{O}(\hat{k}_2 - m)] \Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2)}{\sqrt{m^2 + \mathbf{k}_1^2}(\tau_{\mathcal{P}} + i\varepsilon) \sqrt{m^2 + \mathbf{k}'_1{}^2}(\tau_{\mathcal{Q}} - i\varepsilon)} \times \\ & \times \delta^{(4)}(-\mathcal{Q} + k'_1 + k_2 - \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}) \delta^{(4)}(\mathcal{P} - k_1 - k_2 + \lambda_{\mathcal{P}}\tau_{\mathcal{P}}) + (1 \leftrightarrow 2). \end{aligned}$$

In order to perform integration with respect to  $\mathbf{k}'_1, \mathbf{k}_1, \tau_{\mathcal{Q}}, \tau_{\mathcal{P}}$  in expression (9), we make, in the integrals with respect to  $\mathbf{k}'_1$  and  $\mathbf{k}_1$ , the pure Lorentz transformations (6)  $\Lambda_{\lambda_{\mathcal{Q}}}^{-1} \mathcal{Q} = (M_{\mathcal{Q}}; \mathbf{0})$  and  $\Lambda_{\lambda_{\mathcal{P}}}^{-1} \mathcal{P} = (M_{\mathcal{P}}; \mathbf{0})$ , and consider that  $\mathcal{Q}k_2 = M_{\mathcal{Q}}\Delta_{k_2, m\lambda_{\mathcal{Q}}}^0$  and  $\mathcal{P}k_2 = M_{\mathcal{P}}\Delta_{k_2, m\lambda_{\mathcal{P}}}^0$ ; the measures  $d\Omega_{\mathbf{k}_i} = m d\mathbf{k}_i / \Delta_{k_i, m\lambda_{\mathcal{Q}}}^0$ ,  $i = 1, 2$ , and the delta functions in (9) are invariant under the Lorentz transformations  $\Lambda_{\lambda_{\mathcal{Q}(\mathcal{P})}}^{-1}$  (6). We recast expression (9) into the form

$$(M_Q = M_P = M)$$

$$\begin{aligned} \langle \mathcal{P} | J | \mathcal{Q} \rangle = & \frac{z_1 + z_2}{(2\pi)^3} \int d\Omega_{\Delta_{k,m\lambda_Q}} \Psi_M^*(\Delta_{k,m\lambda_P}) \times \\ & \times \hat{B}(\Delta_{k,m\lambda_P}^0, \Delta_{k,m\lambda_Q}^0) \Psi_M(\Delta_{k,m\lambda_Q}), \end{aligned} \quad (10)$$

where we have defined the wave function for the system in momentum space as

$$\begin{aligned} \Psi_{M_Q}(\Delta_{k,m\lambda_Q}) &= \frac{\Gamma_{M_Q}(\Delta_{k,m\lambda_Q})}{2^{3/2} \sqrt{m} \Delta_{k,m\lambda_Q}^0 (M_Q - 2\Delta_{k,m\lambda_Q}^0 + i\varepsilon)}, \\ \Delta_{k,m\lambda_Q} &= \Delta_{k_2,m\lambda_Q} = -\Delta_{k'_1,m\lambda_Q}, \quad \Delta_{k_2,m\lambda_P} = -\Delta_{k_1,m\lambda_P}, \end{aligned}$$

and for the matrixes  $\hat{O} = \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu$  ( $\mu = 0, 1, 2, 3$ ) under the Lorentz transformations  $\Lambda_{\lambda_Q(\mathcal{P})}^{-1}$  (6) we find

$$\begin{aligned}
 & -\text{Tr} \left[ \hat{O}^+ (\hat{k}_1 + m) (\hat{k}'_1 + m) \hat{O} (\hat{k}_2 - m) \right] = \\
 & = 4m \left( \tilde{a} k_1 k'_1 + \tilde{b} k'_1 k_2 + \tilde{b} k_1 k_2 + \tilde{a} m^2 \right) = \\
 & = \hat{B}(\Delta_{k,m\lambda_{\mathcal{P}}}^0, \Delta_{k,m\lambda_{\mathcal{Q}}}^0) = 2m \left\{ 2\tilde{a} (2\Delta_{k,m\lambda_{\mathcal{P}}}^0) (2\Delta_{k,m\lambda_{\mathcal{Q}}}^0) - \right. \\
 & \quad \left. - (\tilde{a} - \tilde{b}) \left[ (2\Delta_{k,m\lambda_{\mathcal{P}}}^0)^2 + (2\Delta_{k,m\lambda_{\mathcal{Q}}}^0)^2 \right] + 4m^2 (\tilde{a} - \tilde{b}) \right\}, \\
 \tilde{a} = & \begin{cases} 1 & , \hat{O} = \gamma_5 \text{ (pseudoscalar);} \\ 4 & , \hat{O} = \gamma_\mu \text{ (vector);} \\ -4 & , \hat{O} = \gamma_5 \gamma_\mu \text{ (pseudovector),} \end{cases} \quad \tilde{b} = \begin{cases} 1 & , \hat{O} = \gamma_5; \\ 2 & , \hat{O} = \gamma_\mu; \\ 2 & , \hat{O} = \gamma_5 \gamma_\mu. \end{cases}
 \end{aligned}$$

The vector  $\Delta_{k,m\lambda_{\mathcal{P}}}$  from the Lobachevsky space

$$\Delta_{k,m\lambda_{\mathcal{P}}} = \mathbf{k}(-)m\lambda_{\mathcal{P}} = \Lambda_{\lambda_{\mathcal{P}}}^{-1} \mathbf{k} = V(\Lambda_{\lambda_{\mathcal{Q}}}, \mathcal{P}) \Delta_{k,m\lambda_{\mathcal{Q}}}(-) \frac{m}{M} \Delta_{\mathcal{P},\mathcal{Q}},$$

$V(\Lambda_{\lambda_{\mathcal{Q}}}, \mathcal{P}) = \Lambda_{\lambda_{\mathcal{P}}}^{-1} \Lambda_{\lambda_{\mathcal{Q}}} \Lambda_{\Delta_{\mathcal{P},\mathcal{Q}}}$  is Wigner's rotation matrix and  $\Delta_{\mathcal{P},\mathcal{Q}} = \Lambda_{\lambda_{\mathcal{Q}}}^{-1} \mathcal{P}$  is the 4-momentum transfer:

$$\Delta_{\mathcal{P},\mathcal{Q}} = \Lambda_{\mathcal{Q}}^{-1} \mathcal{P} = \mathcal{P} - \frac{\mathcal{Q}}{M} \left( \mathcal{P}_0 - \frac{\mathcal{P} \cdot \mathcal{Q}}{Q_0 + M} \right) = M \sinh \chi_{\Delta} \mathbf{n}_{\Delta},$$

$$\Delta_{\mathcal{P},\mathcal{Q}}^0 = (\Lambda_{\mathcal{Q}}^{-1} \mathcal{P})^0 = \frac{\mathcal{P}_0 Q_0 - \mathcal{P} \cdot \mathcal{Q}}{M} = \frac{\mathcal{P} Q}{M} = M \cosh \chi_{\Delta},$$

$$\mathcal{P} = M \sinh \chi_{\mathcal{P}} \mathbf{n}_{\mathcal{P}}, \quad \mathcal{Q} = M \sinh \chi_{\mathcal{Q}} \mathbf{n}_{\mathcal{Q}}, \quad \mathcal{P}_0 = M \cosh \chi_{\mathcal{P}},$$

$$Q_0 = M \cosh \chi_{\mathcal{Q}}, \quad |\mathbf{n}_{\mathcal{P}}| = |\mathbf{n}_{\mathcal{Q}}| = |\mathbf{n}_{\Delta}| = 1, \quad \Delta^{02}_{\mathcal{P},\mathcal{Q}} - \Delta^2_{\mathcal{P},\mathcal{Q}} = M^2,$$

$\chi_{\Delta}, \chi_{\mathcal{P}}, \chi_{\mathcal{Q}}$  are the respective rapidities, and

$$Q^2 = -t = -2M^2 + 2M \sqrt{M^2 + \Delta^2_{\mathcal{P},\mathcal{Q}}} = 2M^2 (\cosh \chi_{\Delta} - 1).$$

The elastic form factor  $F(t)$  in the case of scalar current we define as

$$F(t) = \langle \mathcal{P} | J | \mathcal{Q} \rangle . \quad (11)$$

It follows that one can consider the elastic form factor  $F(t)$  as a function of the invariant variable  $\Delta_{\mathcal{P},\mathcal{Q}}^2$ , which is the square of the 3-momentum transfer in the Lobachevsky space. Taking into consideration expressions (10)–(11), the form factor  $F(t)$  then represents a convolution of covariant wave RQP-functions in this space. Consequently, by using transformations

$$\psi_{M_{\mathcal{Q}}}(\mathbf{r}) = \frac{1}{(2\pi)^3} \int d\Omega_{\Delta_{p,m\lambda_{\mathcal{Q}}}} \xi(\Delta_{p,m\lambda_{\mathcal{Q}}}, \mathbf{r}) \Psi_{M_{\mathcal{Q}}}(\Delta_{p,m\lambda_{\mathcal{Q}}}),$$

$$\Psi_{M_{\mathcal{Q}}}(\Delta_{p,m\lambda_{\mathcal{Q}}}) = \int d\mathbf{r} \xi^*(\Delta_{p,m\lambda_{\mathcal{Q}}}, \mathbf{r}) \psi_{M_{\mathcal{Q}}}(\mathbf{r}),$$

$$\xi(\Delta_{p,m\lambda_Q}, \mathbf{r}) = \left( \frac{\Delta_{p,m\lambda_Q}^0 - \Delta_{p,m\lambda_Q} \cdot \mathbf{n}}{m} \right)^{-1-ir/\lambda},$$

the completeness condition

$$\frac{1}{(2\pi)^3} \int d\Omega_{\Delta_{p,m\lambda_Q}} \xi(\Delta_{p,m\lambda_Q}, \mathbf{r}) \xi^*(\Delta_{p,m\lambda_Q}, \mathbf{r}') = \delta(\mathbf{r}' - \mathbf{r}),$$

the equation

$$\left( 2\Delta_{p,m\lambda_Q}^0 - \hat{H}_0 \right) \xi(\Delta_{p,m\lambda_Q}, \mathbf{r}) = 0, \quad (12)$$

the addition theorem for relativistic plane waves in the form

$$\int d\omega_n \xi\left(\Delta_{k,m\lambda_Q}(-) \frac{m}{M_Q} \Delta_{\mathcal{P},Q}, \mathbf{r}\right) = \int d\omega_n \xi(\Delta_{k,m\lambda_Q}, \mathbf{r}) \xi^*\left(\frac{m}{M_Q} \Delta_{\mathcal{P},Q}, \mathbf{r}\right),$$

and the fact that the free-Hamiltonian operator (2) is Hermitian, we can represent the form factor (10) as the relativistic Fourier transform of the covariant wave RQP-functions in the configuration

representation:

$$F(t) = 8m^3(z_1 + z_2) \int d\mathbf{r} \xi^* \left( \frac{m}{M} \Delta_{\mathcal{P}, \mathcal{Q}}, \mathbf{r} \right) \left\{ 2\tilde{a} \left| \frac{\hat{H}_0}{2m} \psi_M(\mathbf{r}) \right|^2 - \right. \quad (13)$$

$$\left. -2(\tilde{a} - \tilde{b}) \operatorname{Re} \left[ \psi_M^*(\mathbf{r}) \left( \frac{\hat{H}_0}{2m} \right)^2 \psi_M(\mathbf{r}) \right] + (\tilde{a} - \tilde{b}) |\psi_M(\mathbf{r})|^2 \right\}.$$

For  $s$ -state of the composite system the integrations in (13) respecting of angles gives

$$F_{\ell=0}(t) = 32m^2 \pi(z_1 + z_2) \frac{\chi_\Delta}{\sinh \chi_\Delta} \int_0^\infty d\rho \frac{\sin(\rho\chi_\Delta)}{\rho\chi_\Delta} \left\{ 2\tilde{a} \left| \hat{H}_{0,\ell=0}^{\text{rad}} \varphi_0(\rho, \chi) \right|^2 - \right. \quad (14)$$

$$\left. -2(\tilde{a} - \tilde{b}) \operatorname{Re} \left[ \varphi_0^*(\rho, \chi) \left( \hat{H}_{0,\ell=0}^{\text{rad}} \right)^2 \varphi_0(\rho, \chi) \right] + (\tilde{a} - \tilde{b}) |\varphi_0(\rho, \chi)|^2 \right\}.$$

**For the vector-current case**, covariant expressions for the components of the elastic form factor for the composite system formed by two equal-mass relativistic fermions will have the form

$$F^{(+)}(t) = \frac{16m^4(z_1 + z_2)(2M^2 - t)}{M(4M^2 - t)} \int d\mathbf{r} \xi^* \left( \frac{m}{M} \Delta_{\mathcal{P}, \mathcal{Q}, \mathbf{r}} \right) \times \quad (15)$$

$$\times \text{Re} \left\{ (\tilde{a} - \tilde{b}) \psi_M^*(\mathbf{r}) \frac{\hat{H}_0}{2m} \psi_M(\mathbf{r}) + (\tilde{a} + \tilde{b}) \left[ \frac{\hat{H}_0}{2m} \psi_M(\mathbf{r}) \right]^* \left( \frac{\hat{H}_0}{2m} \right)^2 \psi_M(\mathbf{r}) - \right. \\ \left. - (\tilde{a} - \tilde{b}) \psi_M^*(\mathbf{r}) \left( \frac{\hat{H}_0}{2m} \right)^3 \psi_M(\mathbf{r}) \right\},$$

$$F^{(-)}(t) = \frac{16m^4(z_1 + z_2)(2M^2 - t)}{-Mt} \int d\mathbf{r} \xi^* \left( \frac{m}{M} \Delta_{\mathcal{P}, \mathcal{Q}, \mathbf{r}} \right) \times \quad (16)$$

$$\times \text{Im} \left\{ (\tilde{a} - \tilde{b}) \psi_M(\mathbf{r}) \left[ \frac{\hat{H}_0}{2m} \psi_M(\mathbf{r}) \right]^* + (3\tilde{a} - \tilde{b}) \left[ \frac{\hat{H}_0}{2m} \psi_M(\mathbf{r}) \right] \times \right. \\ \left. \times \left[ \left( \frac{\hat{H}_0}{2m} \right)^2 \psi_M(\mathbf{r}) \right]^* - (\tilde{a} - \tilde{b}) \psi_M(\mathbf{r}) \left[ \left( \frac{\hat{H}_0}{2m} \right)^3 \psi_M(\mathbf{r}) \right]^* \right\},$$



where the transverse component of the elastic form factor in (16) for real-valued potentials does not vanish even at equal masses.

For  $s$ -state of the composite system the integrations in (15) and (16) respecting of angles gives

$$F_{\ell=0}^{(+)}(t) = \frac{64\pi m^3(z_1 + z_2)(2M^2 - t)}{M(4M^2 - t)} \frac{\chi_\Delta}{\sinh \chi_\Delta} \int_0^\infty d\rho \frac{\sin(\rho\chi_\Delta)}{\rho\chi_\Delta} \times \quad (17)$$

$$\times \text{Re} \left\{ (\tilde{a} - \tilde{b}) \varphi_0^*(\rho, \chi) \hat{H}_{0,\ell=0}^{\text{rad}} \varphi_0(\rho, \chi) + (\tilde{a} + \tilde{b}) \left[ \hat{H}_{0,\ell=0}^{\text{rad}} \varphi_0(\rho, \chi) \right]^* \times \right.$$

$$\left. \times \left( \hat{H}_{0,\ell=0}^{\text{rad}} \right)^2 \varphi_0(\rho, \chi) - (\tilde{a} - \tilde{b}) \varphi_0^*(\rho, \chi) \left( \hat{H}_{0,\ell=0}^{\text{rad}} \right)^3 \varphi_0(\rho, \chi) \right\},$$

$$\begin{aligned}
F_{\ell=0}^{(-)}(t) &= \frac{64\pi m^3(z_1 + z_2)(2M^2 - t)}{-Mt} \frac{\chi_\Delta}{\sinh \chi_\Delta} \int_0^\infty d\rho \frac{\sin(\rho\chi_\Delta)}{\rho\chi_\Delta} \times \quad (18) \\
&\times \text{Im} \left\{ (\tilde{a} - \tilde{b})\varphi_0(\rho, \chi) [\hat{H}_{0,\ell=0}^{\text{rad}}\varphi_0(\rho, \chi)]^* + (3\tilde{a} - \tilde{b}) [(\hat{H}_{0,\ell=0}^{\text{rad}})^2\varphi_0(\rho, \chi)]^* \times \right. \\
&\quad \left. \times \hat{H}_{0,\ell=0}^{\text{rad}}\varphi_0(\rho, \chi) - (\tilde{a} - \tilde{b})\varphi_0(\rho, \chi) [(\hat{H}_{0,\ell=0}^{\text{rad}})^3\varphi_0(\rho, \chi)]^* \right\},
\end{aligned}$$

where the transverse component (18) of the form factor for real-valued potentials vanishes.

#### 4. Root-mean-square radius and form factor for Coulomb interaction

The expression for the invariant root-mean-square radius (r.m.s.) of a composite system in terms of the wave function  $s$ -state in the case of scalar current (14) has form

$$\langle r_{0,S}^2 \rangle = \frac{6\partial F_{\ell=0}(t)/\partial t|_{t=0}}{F_{\ell=0}(0)} = \frac{1}{M^2} + \frac{1}{M^2} \frac{\int_0^\infty d\rho \rho^2 R_S(\rho)}{\int_0^\infty d\rho R_S(\rho)}, \quad (19)$$

$$R_S(\rho) = 2\tilde{a} |\hat{H}_{0,\ell=0}^{\text{rad}} \varphi_0(\rho, \chi)|^2 - 2(\tilde{a} - \tilde{b}) \text{Re}[\varphi_0^*(\rho, \chi) (\hat{H}_{0,\ell=0}^{\text{rad}})^2 \varphi_0(\rho, \chi)] + (\tilde{a} - \tilde{b}) |\varphi_0(\rho, \chi)|^2.$$

Thus, if function  $R_S(\rho)$  has a constant sign then the wave function of  $s$ -state describes not all structure of the composite particle, but

only the region which be upon distances that larger its of the Compton wavelength  $1/M$ . The central sphere with  $\langle r_{0,S}^2 \rangle = 1/M^2$  will correspond function of spatial distribution in the form  $R_S(\rho) = \delta(\rho)/4\pi$ . This distribution brings about the value of contribution to form factor from this sphere that equal

$$F_{\ell=0}(Q^2)|_{r_{0,S}=1/M} = 8m^2(z_1 + z_2) \frac{\chi_\Delta}{\sinh \chi_\Delta}.$$

But if  $R_S(\rho)$  in (27) has not a constant sign then negative importances

$$\langle r_S^2 \rangle = \frac{1}{M^2} \int_0^\infty d\rho \rho^2 R_S(\rho) \left[ \int_0^\infty d\rho R_S(\rho) \right]^{-1}$$

answer on this. So contribution negative  $\langle r_S^2 \rangle$  brings, as this is seen from (19), to decrease of the value  $\langle r_{0,S}^2 \rangle$  meson in contrast with its of the Compton wavelength  $1/M$ . This result conform with experimental importance for  $\pi$ -meson.

The result for the vector-current case obtained in terms of the longitudinal form factor component in (17) has the form

$$\langle r_{0,V}^2 \rangle = \frac{1}{M^2} \left[ 1 + \int_0^\infty d\rho \left( \rho^2 - \frac{3}{2} \right) R_V(\rho) \left( \int_0^\infty d\rho R_V(\rho) \right)^{-1} \right], \quad (20)$$

$$\begin{aligned} R_V(\rho) = & \operatorname{Re} \left\{ (\tilde{a} - \tilde{b}) \varphi_0^*(\rho, \chi) \hat{H}_{0,\ell=0}^{\text{rad}} \varphi_0(\rho, \chi) + \right. \\ & + (\tilde{a} + \tilde{b}) \left[ \hat{H}_{0,\ell=0}^{\text{rad}} \varphi_0(\rho, \chi) \right]^* \left( \hat{H}_{0,\ell=0}^{\text{rad}} \right)^2 \varphi_0(\rho, \chi) - \\ & \left. - (\tilde{a} - \tilde{b}) \varphi_0^*(\rho, \chi) \left( \hat{H}_{0,\ell=0}^{\text{rad}} \right)^3 \varphi_0(\rho, \chi) \right\}. \end{aligned} \quad (21)$$

Thus, if the function  $R_V(\rho)$  is sign-constant and if

$\int_0^\infty d\rho \left( \rho^2 - 3/2 \right) R_V(\rho) > 0$ , then it describes only the region lying at distances longer than the Compton wavelength  $1/M$  of the composite particle.

In accordance with Eq. (17) the contribution of central sphere with  $\langle r_{0,V}^2 \rangle = 1/M^2$  to the form factor has the form

$$F_{\ell=0}^{(+)}(t)|_{r_{0,V}=1/M} = \frac{16m^3(z_1 + z_2)(2M^2 - t)}{M(4M^2 - t)} \frac{\chi_\Delta}{\sinh \chi_\Delta}.$$

If  $R_V(\rho)$  is sign-constant, but  $\int_0^\infty d\rho (\rho^2 - 3/2) R_V(\rho) < 0$ , then negative values will correspond to it,

$$\langle r_V^2 \rangle = \frac{1}{M^2} \int_0^\infty d\rho \left( \rho^2 - \frac{3}{2} \right) R_V(\rho) \left( \int_0^\infty d\rho R_V(\rho) \right)^{-1}.$$

$\langle r_V^2 \rangle$  also takes negative values if the  $R_V(\rho)$  in Eq. (21) is not sign-constant. In either case, the contribution of negative values of  $\langle r_V^2 \rangle$  leads, as can be seen from Eq. (20), to a decrease in the r.m.s.  $\langle r_{0,V}^2 \rangle$  of the meson in relation to its Compton wavelength  $1/M$ . This result agrees with the experimental value for the pion.

As examples, we consider the form factors of meson in the case of the Coulomb field

$$V(r) = -\frac{\alpha_s}{r}, \alpha_s > 0. \quad (22)$$

The radial wave function of exact solution of the RQP-equation (4) with interaction (22) for the  $s$ -state and ground level  $n = 1$  with the energy  $M_1$  has the form [Yu. D. Chernichenko, *Phys. At. Nucl.* **80**, 707 (2017)],

$$\varphi_0^{(1)}(\rho, \kappa_1) = C_0^{(1)}(\kappa_1)(\rho - \rho_{\kappa_1})e^{(\rho - \rho_{\kappa_1})\kappa_1}, \quad (23)$$

where

$$\rho_{\kappa_1} = \frac{a\tilde{\alpha}_s}{2} \cos \kappa_1, M_1 = 2m \cos \kappa_1, 0 < \kappa_1 < \pi/2, \tilde{\alpha}_s = m\alpha_s,$$

and  $\kappa_1$  defines by the following quantization condition

$$\tilde{\alpha}_s(a \cos^2 \kappa_1 + b) = 4 \sin \kappa_1. \quad (24)$$

The normalization factor

$|C_0^{(1)}(\kappa_1)|^2 = m\kappa_1^3 e^{-2\kappa_1\rho_{\kappa_1}} / \pi(2\kappa_1^2\rho_{\kappa_1}^2 - 2\kappa_1\rho_{\kappa_1} + 1)$  and it can be found from the normalization condition

$$4\pi \int_0^\infty dr |\varphi_0^{(1)}(r, \kappa_1)|^2 = 1.$$

Then the form factor (14) and the r.m.s. (19) with interaction (22) for the ground level of  $s$ -state are given as

$$F_{\ell=0, n=1}^{\text{Coul}}(Q^2) = \frac{64(z_1 + z_2)m^3\kappa_1^3}{(2\kappa_1^2\rho_{\kappa_1}^2 - 2\kappa_1\rho_{\kappa_1} + 1)} \frac{\chi_\Delta}{\sinh \chi_\Delta} \left\{ \frac{2\kappa_1 A_S}{(\chi_\Delta^2 + 4\kappa_1^2)^2} - \right. \quad (25)$$

$$\left. - \frac{2B_S}{\chi_\Delta^2 + 4\kappa_1^2} + \frac{C_S}{\chi_\Delta} \arctan \frac{\chi_\Delta}{2\kappa_1} \right\},$$

$$\langle r_{0,S}^2 \rangle_{\ell=0, n=1}^{\text{Coul}} = \frac{1}{M_1^2} \left\{ 1 + \frac{1}{2\kappa_1^2} \left[ 1 + \frac{5A_S - 8\kappa_1 B_S}{A_S - 4\kappa_1 B_S + 4\kappa_1^2 C_S} \right] \right\}, \quad (26)$$

$$A_S = \tilde{a} - \tilde{b} + 2\tilde{b} \cos^2 \kappa_1, \quad B_S = \tilde{b} \cos \kappa_1 \sin \kappa_1, \quad C_S = \tilde{a} \sin^2 \kappa_1.$$



For large  $Q^2 = -t$  the rapidity behaves as  $\chi_\Delta \approx \ln(Q^2/M_1^2)$  and, consequently, the leading behavior of form factor (25) gives by expression [Yu. D. Chernichenko, Phys. At. Nucl. **81**, 360 (2018)]

$$F_{\ell=0, n=1}^{\text{Coul}}(Q^2) \approx \frac{64\pi(z_1 + z_2)m^3\kappa_1^3 C_S}{2\kappa_1^2\rho_{\kappa_1}^2 - 2\kappa_1\rho_{\kappa_1} + 1} \frac{1}{(Q/M_1)^2} \left\{ 1 + O\left[\ln^{-1}(Q/M_1)^2\right] \right\}.$$

Such behavior of the form factor under large  $t$  complies with the prediction of the dimensional quark counting rule [V. A. Matveev, R. M. Muradyan, A. N. Tavkhelidze, Lett. Nuovo Cimento **5**, 907 (1972); **7**, 719 (1973); Theor. Math. Phys. **40**, 778 (1979); S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973)], which gives  $F_\pi \sim |t|^{-1}$ . In the case of relativistic spinless particles the decrease of the form factor occurs under the law  $F_\pi \sim (|t| \ln^3 |t|)^{-1}$  [Yu. D. Chernichenko, Phys. At. Nucl. **77**, 229 (2014)].

The result for the longitudinal form factor component (17) and the r.m.s. (20) of the two-fermion composite system with the chromodynamic interaction (22) for the ground level of  $s$ -state will have the form [Yu. D. Chernichenko, *Phys. At. Nucl.* **82**, 334 (2019)]:

$$\begin{aligned}
 F_{\ell=0,n=1}^{(+)\text{Coul}}(t) &= \frac{64(z_1 + z_2)m^4\kappa_1^3 \cos \kappa_1 (2M_1^2 - t)}{(2\kappa_1^2\rho_{\kappa_1}^2 - 2\kappa_1\rho_{\kappa_1} + 1)M_1(4M_1^2 - t)} \frac{\chi_\Delta}{\sinh \chi_\Delta} \times \quad (27) \\
 &\times \left\{ \frac{4\kappa_1 A_S}{(\chi_\Delta^2 + 4\kappa_1^2)^2} + \frac{B_V}{\chi_\Delta^2 + 4\kappa_1^2} + \frac{2C_V}{\chi_\Delta} \arctan \frac{\chi_\Delta}{2\kappa_1} \right\}, \\
 \langle r_{0,V}^2 \rangle_{\ell=0,n=1}^{(+)\text{Coul}} &= \frac{1}{M_1^2} \left\{ -\frac{1}{2} + \frac{1}{2\kappa_1^2} \left[ 1 + \frac{5A_S + 2\kappa_1 B_V}{A_S + \kappa_1 B_V + 4\kappa_1^2 C_V} \right] \right\}, \\
 B_V &= \tan \kappa_1 (\tilde{a} - \tilde{b} - 2\tilde{b} \cos^2 \kappa_1), \quad C_V = (\tilde{a} - \tilde{b}) \sin^2 \kappa_1.
 \end{aligned}$$

At reception of the expressions (27), we excluded  $\tilde{\alpha}_s$  by means of not only the quantization condition (24) but also the identity

$$a(\tilde{a} + \tilde{b}) - 2\tilde{b}(a + b) \equiv 0. \quad (28)$$

For large  $Q^2$ , the behavior of the form factor in (27) gives as

$$F_{\ell=0,n=1}^{(+)\text{Coul}}(t) \approx \frac{128(z_1 + z_2)m^4 \kappa_1^3 \cos \kappa_1}{M_1(2\kappa_1^2 \rho_{\kappa_1}^2 - 2\kappa_1 \rho_{\kappa_1} + 1)} \frac{1}{(Q/M_1)^2} \times \quad (29)$$

$$\times [\pi C_V + B_V \ln^{-1}(Q/M_1)^2].$$

From the asymptotic expression (29), one can see that the behavior of the form factor for large  $t = -Q^2$  also obeys the  $F(t) \sim |t|^{-1}$  law [Yu. D. Chernichenko, *Phys. At. Nucl.* **82**, 334 (2019)] predicted by the dimensional-quark-counting rule [V. A. Matveev, R. M. Muradyan, A. N. Tavkhelidze, *Lett. Nuovo Cimento* **5**, 907 (1972); **7**, 719 (1973); *Theor. Math. Phys.* **40**, 778 (1979); S. J. Brodsky and G. R. Farrar, *Phys. Rev. Lett.* **31**, 1153 (1973)].

Thus, exactly registration of the spin brings about such behaviour of the form factor for large  $t$  which is predicted by the dimensional-quark-counting rule.

By using the results that were obtained here, we can calculate the r.m.s. (26) for the pions  $\pi^\pm$  (pseudoscalar) with  $n = 1, \ell = 0, a = 1, b = 0, \tilde{a} = \tilde{b} = 1, \tilde{\alpha}_s \cos \kappa_1 = 4 \tan \kappa_1$ , the energy  $M_1 = M_{\pi^\pm} = 0.140 \text{ GeV}$  [K. A. Olive et al. (Particle Data Group), *Chin. Phys. C* **38** (2014)], and the Coulomb wave function (23). Expression (26) can then be represented to the form

$$\langle r_{0,S}^2 \rangle_{\ell=0,n=1}^{\text{Coul},\pi^\pm} = \frac{0.0389}{M_1^2} \left\{ 1 + \frac{1}{2\kappa_1^2} \left[ 1 + \frac{6 - 4(2\kappa_1 \tan \kappa_1 - 1)}{(2\kappa_1 \tan \kappa_1 - 1)^2 + 1} \right] \right\} \text{ fm}^2. \quad (30)$$

Calculation of the expression (30) with energy pions

$M_1 = M_{\pi^\pm} = 0.140 \text{ GeV}$  and importance of the rapidity

$\kappa_1 = 1.1414$  gives:

$\langle r_{0,S}^2 \rangle_{\ell=0,n=1}^{\text{Coul},\pi^\pm} = 2.30 \text{ fm}^2 \gg \langle r_0^2 \rangle_{\text{exp}}^{\pi^\pm} = 0.45 \pm 0.01 \text{ fm}^2$  [K. A. Olive et al. (Particle Data Group), *Chin. Phys. C* **38** (2014)]. For this

importance of the rapidity  $\kappa_1 = 1.1414$  the parameters to model (the coupling constant and mass quark) have importances:

$\tilde{\alpha}_s = 20.98, m = 0.168 \text{ GeV}$ .

For kaons with energy  $M_1 = M_{K^\pm} = 0.494 \text{ GeV}$  and importance of the rapidity  $\kappa_1 = 0.8902$  we have:  $\langle r_{0,S}^2 \rangle_{\ell=0,n=1}^{\text{Coul},K^\pm} = 0.31 \text{ fm}^2$  that complies with  $\langle r_0^2 \rangle_{\text{exp}}^{K^\pm} = 0.31 \pm 0.03 \text{ fm}^2$  [K. A. Olive et al. (Particle Data Group), *Chin. Phys. C* **38** (2014)]. For importance of the rapidity  $\kappa_1 = 0.8902$  the parameters to model have importances:  $\tilde{\alpha}_s = 7.85, m = 0.393 \text{ GeV}$ .

The r.m.s. for vector-current case (34) of the  $\pi^\pm$  mesons with  $n = 1, \ell = 0, a = 1, b = 0, \tilde{a} = \tilde{b} = 1, \tilde{\alpha}_s \cos \kappa_1 = 4 \tan \kappa_1, M_1 = 0.140 \text{ GeV}$  and the Coulomb wave function (23) reduces to the form

$$\langle r_{0,V}^2 \rangle_{\ell=0,n=1}^{(+)\text{Coul},\pi^\pm} = \frac{0.0389}{M_1^2} \left\{ -\frac{1}{2} + \frac{3}{2\kappa_1^2} \left[ 1 - \frac{1}{\kappa_1 \tan \kappa_1 - 1} \right] \right\} \text{ fm}^2, \quad (31)$$

which for  $\kappa_1 = 1.3687$  gives  $\langle r_{0,V}^2 \rangle_{\ell=0,n=1}^{(+)\text{Coul},\pi^\pm} = 0.32 \text{ fm}^2$  with  $\tilde{\alpha}_s = 97.26, m = 0.349 \text{ GeV}$ , and it lies near the confidence interval of its experimental value,  $\langle r_0^2 \rangle_{\text{exp}}^{\pi^\pm} = 0.45 \pm 0.01 \text{ fm}^2$  [K. A. Olive et al. (Particle Data Group), *Chin. Phys. C* **38** (2014)].

For kaons with energy  $M_1 = M_{K^\pm} = 0.494 \text{ GeV}$  and importance of the rapidity  $\kappa_1 = 1.3687$  we have:  $\langle r_{0,V}^2 \rangle_{\ell=0,n=1}^{\text{Coul},K^\pm} = 0.03 \text{ fm}^2$  that much smaller than  $\langle r_0^2 \rangle_{\text{exp}}^{K^\pm} = 0.31 \pm 0.03 \text{ fm}^2$ . For importance of the rapidity  $\kappa_1 = 1.3687$  the parameters to model have importances:  $\tilde{\alpha}_s = 97.26, m = 1.231 \text{ GeV}$ .

Thus, the value r.m.s. of charged pions that has been obtained here corresponds the case of vector-current, but the value r.m.s. of charged kaons corresponds the case of scalar-current.

This distinction means that the quark–quark potential for pseudoscalar mesons should include, in addition to Coulomb interaction, the confining and spin–spin components and take into account the difference in the quark masses.

It should be emphasized that expression (31) for the r.m.s. has a singularity at  $\kappa_1 \tan \kappa_1 = 1$  that is, at  $\kappa_1 \simeq 0.86034$ . The same singularity arose in the scalar-current case [Yu. D. Chernichenko, *Phys. At. Nucl.* **81**, 360 (2018)].

## 5. Conclusions

For the scalar and the vector-current cases, new covariant expressions of the elastic form factor for the composite system formed by two relativistic spin  $1/2$  particles with equal masses has been found as functions of the invariant variable  $\Delta_{\mathcal{P},\mathcal{Q}}^2$ , which is the square of the 3-momentum transfer in Lobachevskys space. The expressions for the invariant root-mean-square radius of a composite system in terms of the wave function  $s$ -state were obtained. The pseudoscalar, vector, and pseudovector cases has been considered. The consideration is conducted within the framework of RQP approach on the basis of covariant Hamiltonian formulation of quantum field theory, by transition to the three-dimensional relativistic configurational representation in the case of bound system of two relativistic spin particles with equal masses.



The identity in the form (28) valid for the values of the spin parameters  $a, b, \tilde{a}$ , and  $\tilde{b}$  in the pseudoscalar, vector, and pseudovector cases has been established.

The application of the three-dimensional relativistic configuration representation for a system of two relativistic spin particles with equal masses has allowed to install that if the function of spatial distribution has a constant sign then the wave function of  $s$ -state describes not all structure of the composite particle, but only the region which be upon distances that larger its of the Compton wavelength  $1/M$ . But if the function of spatial distribution has not a constant sign then this brings to decrease of the value  $\langle r_0^2 \rangle$  meson in contrast with its of the Compton wavelength  $1/M$ . This result conform with experimental importance for mesons.

The present analysis has shown that a dominant contribution to the structure of a composite system from the central sphere of radius  $1/M$  is proportional to  $\chi_\Delta / \sinh \chi_\Delta$ . In the nonrelativistic limit, this relativistic geometric factor tends to unity.

As examples, for the scalar and the vector-current cases the expressions for the form factor of a bound system formed by two relativistic spin  $1/2$  particles with equal masses in the case of Coulomb potential were obtained. It is installed that the covariant wave RQP-function of Coulomb potential at  $Q^2 = -t \gg 1$  give the decrease for these form factors under the law  $F_\pi \sim |t|^{-1}$ , which predicts the dimensional quark counting rule.

For the scalar and vector-current cases, we calculated the r.m.s. of the ground level of bound  $s$ -state of charged pions and kaons, and it have conducted comparison with its experimental values.

## Acknowledgments

I am grateful to O. P. Solovtsova and V. V. Andreev for attention to this work, support, and enlightening comments, and to A. E. Dorokhov, Yu. A. Kurochkin, and I. S. Satsunkevich for their interest in this study and valuable discussions on its results. Partial support of the work by the International Program of Cooperation between Republic of Belarus and the Joint Institute of Nuclear Research (JINR, Dubna) and State Program of the scientific studies “Konvergentsiya-2020”.

May 2020

*Thank for Attention*

*Yu. D. Chernichenko, V. V. Kondratjuk*