Pseudoscalar Mesons in a Covariant Constituent Quark Model with Infrared Confinement.

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Covariant Constituent Quark Model with Infrared Confinement

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Lagrangian of the interaction between hadrons and quarks is written as:

$$L_{int}^{st}(x) = g_M M(x) \int dx_1 \int dx_2 \, F_M(x, x_1, x_2) \bar{q}_1(x_1) \lambda_M \Gamma_M q_2(x_2)$$

Where q_i is a member of quark triplet,
$M(x)$ –Euclidean field connected with the fields of physical particles
λ_M and Γ_M are the Gell-Mann and Dirac matrixes
g_M is the quark-meson coupling constant

$F_M(x, x_1, x_2)$-vertex function, characterizing the finite size of the meson

To satisfy translational invariance the vertex function has to obey the identity

$$\boldsymbol{F_M(x+a, x_1+a, x_2+a) = F_M(x, x_1, x_2)}$$

for any vector a.

$$\boldsymbol{F_M(x, x_1, x_2)} = \delta^4\left(x - \sum_{i=1}^{2} w_i x_i\right)\Phi_M\left((x_1 - x_2)^2\right)$$

$$w_i = \frac{m_i}{m_1 + m_2}$$

m_1, m_2- masses of constituent quarks

The simplest choice:

$$\boldsymbol{\widetilde{\Phi}_M(-l^2) = \int dx e^{ilx}\Phi_M(x^2) = \exp\left(\frac{l^2}{\Lambda_M^2}\right)}$$

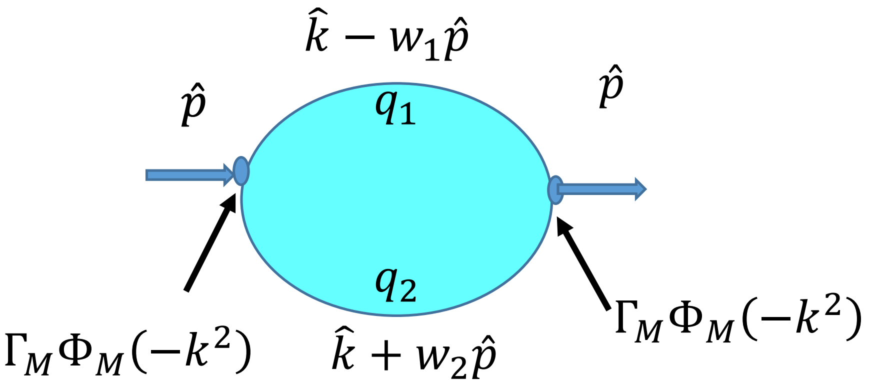
$\boldsymbol{\Lambda_M^2}$-characterizes the size of the meson

$\boldsymbol{g_M}$-the coupling constants for meson-quark interaction are defined from so-called compositeness condition:

$$Z_M = 1 + \frac{3g_M^2}{4\pi^2}\widetilde{\Pi}'_M(m_M^2) = 0$$

It is convenient to use:

$$h_M = \frac{3g_M^2}{4\pi^2}\widetilde{\Pi}'_M(m_M^2) = -\frac{1}{\widetilde{\Pi}'_M(m_M^2)}$$



$$w_{1,2} = \frac{m_{q_{1,2}}}{m_{q_1+}m_{q_2}}$$

$$\Pi_M(p^2) = \int \frac{d^4k}{(2\pi)^2 i} \Phi_M^2(-k^2) Tr\{\Gamma_M \, S_{q_1}(\hat{k} - w_1\hat{p}) \Gamma_M S_{q_2}(\hat{k} + w_2\hat{p})\}$$

$$\Pi_P(p^2) = \int \frac{d^4k}{(2\pi)^2 i} \Phi_P^2(-k^2) Tr\{i\gamma^5 \Gamma^M \, S_{q_1}(\hat{k} - w_1\hat{p}) i\gamma^5 S_{q_2}(\hat{k} + w_2\hat{p})\}$$

$$\Pi_V(p^2) = \frac{1}{3}\left[g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right] \int \frac{d^4k}{(2\pi)^2 i} \Phi_V^2(-k^2) Tr\{\gamma_\mu S_{q_1}(\hat{k} - w_1\hat{p}) \gamma_\nu S_{q_2}(\hat{k} + w_2\hat{p})\}$$

$$S_q(\hat{k}) = \frac{1}{m_q - \hat{k} - i\epsilon}$$ *-free propagator of constituent quark*

Fock-Shwinger representation:

$$S_q(\hat{k} + \hat{p}) = \frac{1}{m_q - \hat{k} - \hat{p}} = \frac{m_q + \hat{k} + \hat{p}}{m_q^2 - (k+p)^2} = (m_q + \hat{k} + \hat{p}) \int_0^\infty d\alpha \, e^{-\alpha\left(m_q^2 - (k+p)^2\right)}$$

The derivative of $\Pi_M(p^2)$ can be calculated by means of using identity:

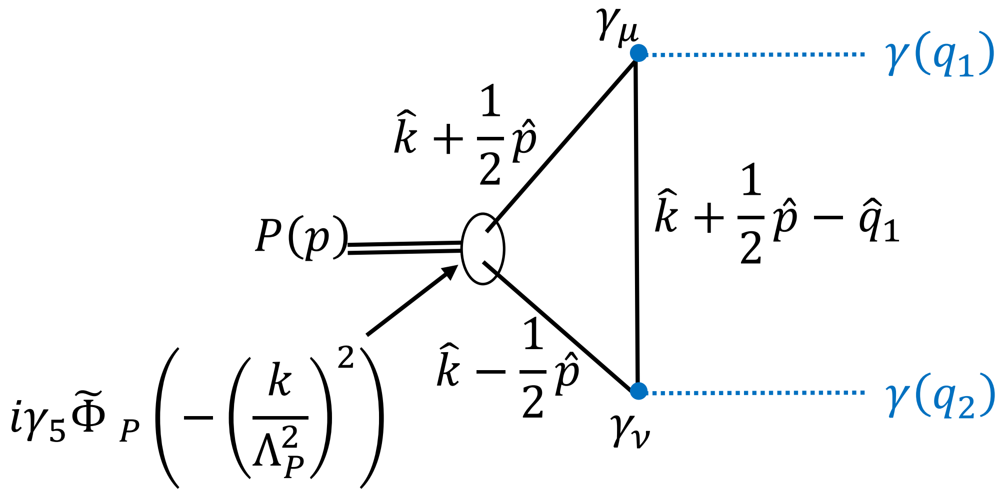
$$\frac{\partial}{\partial p^\mu}\frac{1}{m_q-\hat{k}-\hat{p}}=\frac{1}{m_q-\hat{k}-\hat{p}}\gamma^\mu\frac{1}{m_q-\hat{k}-\hat{p}}$$

$$\tilde{\Pi}'_M(p^2)=\frac{1}{2p^2}p^\alpha\frac{d}{dp^\alpha}\int\frac{d^4k}{(2\pi)^4 i}\Phi_M^2(-k^2)Tr\{\Gamma_M S_{q_1}(\hat{k}-w_1\hat{p})\Gamma_M S_{q_2}(\hat{k}+w_2\hat{p})\}=$$

$$=\frac{1}{2p^2}\int\frac{d^4k}{(2\pi)^4 i}\Phi_M^2(-k^2)\Big[-w_1 Tr\{\Gamma_M S_{q_1}(\hat{k}-w_1\hat{p})\hat{p}S_{q_1}(\hat{k}-w_1\hat{p})\Gamma_M S_{q_2}(\hat{k}+w_2\hat{p})\}+$$

$$+w_2 Tr\left\{\Gamma_M S_{q_1}(\hat{k}-w_1\hat{p})\Gamma_M S_{q_2}(\hat{k}+w_2\hat{p})pS_{q_2}(\widehat{k+w_2}\hat{p})\right\}\Big]$$

$P \to \gamma\gamma$ decay:



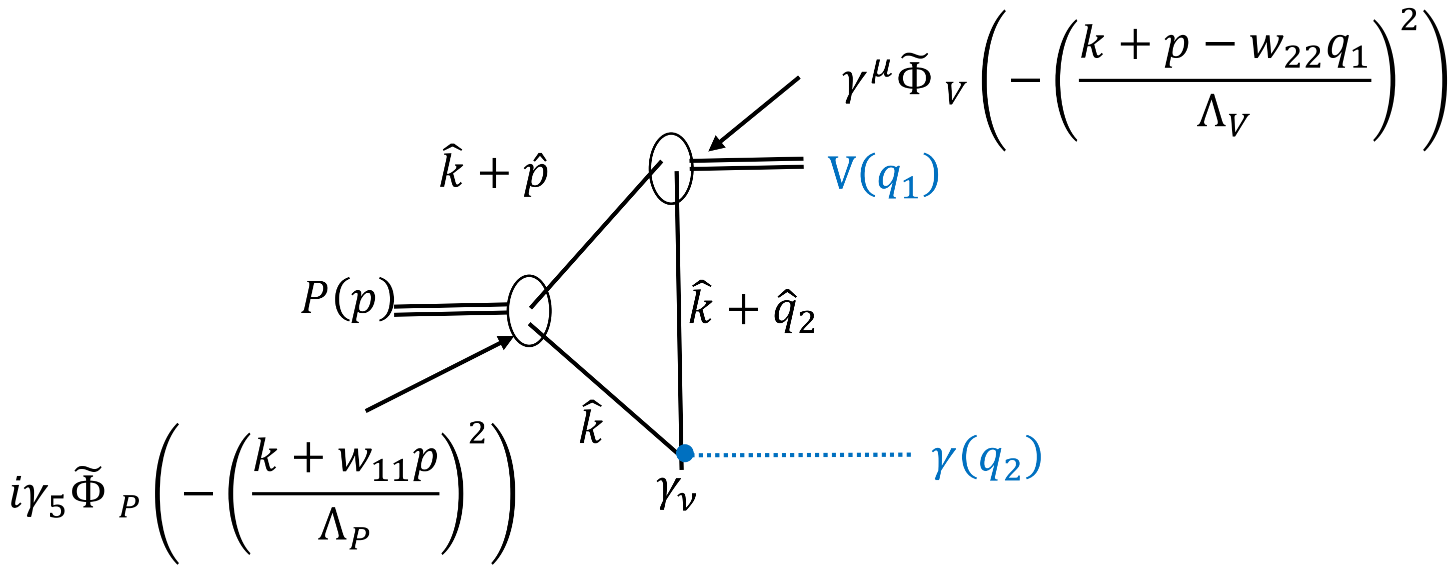
$$= 3g_P Tr\{\lambda_P Q^2\}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left\{i\gamma_5\left(m_q+\hat{k}+\frac{1}{2}\hat{p}\right)\gamma_\mu\left(m_q+\hat{k}+\frac{1}{2}\hat{p}-\hat{q}_1\right)\gamma_\nu\left(m_q+\hat{k}-\frac{1}{2}\hat{p}\right)\right\}e^{\left(\frac{k}{\Lambda_P}\right)^2}\int_0^\infty d\alpha_1\int_0^\infty d\alpha_2\int_0^\infty d\alpha_3$$

$$e^{-\alpha_1\left(m_q^2-\left(k+\frac{1}{2}p\right)^2\right)-\alpha_2\left(m_q^2-\left(k+\frac{1}{2}p-q_1\right)^2\right)-\alpha_3\left(m_q^2-\left(k-\frac{1}{2}p\right)^2\right)} =$$

$$= 3g_P Tr\{\lambda_P Q^2\}\int_0^\infty d\alpha_1\int_0^\infty d\alpha_2\int_0^\infty d\alpha_3\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left\{i\gamma_5\left(m_q+\hat{k}+\frac{1}{2}\hat{p}\right)\gamma_\mu\left(m_q+\hat{k}+\frac{1}{2}\hat{p}-\hat{q}_1\right)\gamma_\nu\left(m_q+\hat{k}-\frac{1}{2}\hat{p}\right)\right\}$$

$$e^{a_{P\gamma\gamma}(\alpha)k^2+2kr_{P\gamma\gamma}(\alpha,p,q_1)-z_{P\gamma\gamma}(\alpha,m_q,p,q_1,q_2)}$$

$P \to V\gamma$ decay:



$$g_{PV\gamma}\varepsilon^{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta = 3g_P g_V Tr\{Q\{\lambda_V,\lambda_P\}\lambda_P\}\int \frac{d^4k}{(2\pi)^4}\widetilde{\Phi}_P\left(-\left(\frac{k+w_{11}p}{\Lambda_P}\right)^2 \widetilde{\Phi}_V\left(-\left(\frac{k+p-w_{22}q_1}{\Lambda_V}\right)^2\right)\right)\times$$

$$\times \operatorname{Tr}\left\{i\gamma_5 \frac{1}{m_q-(\hat{k}+\hat{p})}\gamma_\mu \frac{1}{m_q-(\hat{k}+\hat{q}_2)}\gamma_\nu \frac{1}{m_q-k}\right\} =$$

$$= 3g_P g_V Tr\{Q\{\lambda_V,\lambda_P\}\lambda_P\}\int_0^\infty d\alpha_1 \int_0^\infty d\alpha_2 \int_0^\infty d\alpha_3 \int \frac{d^4k}{(2\pi)^4}\operatorname{Tr}\{i\gamma_5(m_q+\hat{k}+\hat{p})\gamma_\mu(m_q+\hat{k}+\hat{q}_2)\gamma_\nu(m_q+\hat{k})\}$$

$$e^{a_{PV\gamma}(\alpha)k^2+2kr_{PV\gamma}(\alpha,p,q_1)-z_{PV\gamma}(\alpha,m_q,p,q_1,q_2)}$$

$$k^{\mu} e^{ak^2+2kr+z_0} = \frac{1}{2}\frac{\partial}{\partial r^{\mu}} e^{ak^2+2kr+z_0}$$

$$k^{\mu} k^{\nu} e^{ak^2+2kr+z_0} = \frac{1}{2}\frac{\partial}{\partial r^{\mu}}\frac{1}{2}\frac{\partial}{\partial r^{\upsilon}} e^{ak^2+2kr+z_0}$$

$$\mathrm{Tr}\left\{i\gamma_5\left(m_q+\hat{k}+\frac{1}{2}\hat{p}\right)\gamma_\mu\left(m_q+\hat{k}+\frac{1}{2}\hat{p}-\hat{q}_1\right)\gamma_\nu\left(m_q+\hat{k}-\frac{1}{2}\hat{p}\right)\right\}\Rightarrow$$

$$\Rightarrow \mathrm{Tr}\{i\gamma_5(m_q+\gamma^{\rho})\gamma_\mu(m_q+\gamma^{\sigma})\gamma_\nu(m_q+\gamma^{\tau})\}\left(\frac{1}{2}\frac{\partial}{\partial r^{\rho}}+\frac{1}{2}p^{\rho}\right)\left(\frac{1}{2}\frac{\partial}{\partial r^{\sigma}}+\frac{1}{2}p^{\sigma}-q_1^{\sigma}\right)\left(\frac{1}{2}\frac{\partial}{\partial r^{\tau}}-\frac{1}{2}p^{\tau}\right)$$

$$\mathrm{Tr}\{i\gamma_5(m_q+\hat{k}+\hat{p})\gamma_\mu(m_q+\hat{k}+\hat{q}_2)\gamma_\nu(m_q+\hat{k})\}\Rightarrow$$

$$\Rightarrow \mathrm{Tr}\{i\gamma_5(m_q+\gamma^{\rho})\gamma_\mu(m_q+\gamma^{\sigma})\gamma_\nu(m_q+\gamma^{\tau})\}\left(\frac{1}{2}\frac{\partial}{\partial r^{\rho}}+p^{\rho}\right)\left(\frac{1}{2}\frac{\partial}{\partial r^{\sigma}}+\frac{1}{2}q_2^{\sigma}\right)\frac{1}{2}\frac{\partial}{\partial r^{\tau}}$$

$$\int \frac{d^4k}{4\pi^2 i} e^{a(\alpha)k^2+2kr(\alpha,p)-z_0(\alpha,m_q,p)} = \{k_0 = ik_4; k_E^2 \leq 0, p_E^2 \leq 0\} =$$

$$= \frac{1}{a(\alpha)} e^{-\frac{r^2(\alpha,p)}{a(\alpha)}-z_0(\alpha,m_q,p)}$$

$$\frac{\partial}{\partial r^\mu} e^{-\frac{r^2}{a}} = e^{-\frac{r^2}{a}}\left(-\frac{2r^\mu}{a} + \frac{\partial}{\partial r^\mu}\right)$$

$$\frac{\partial}{\partial r^\mu}\frac{\partial}{\partial r^\nu} e^{-\frac{r^2}{a}} = e^{-\frac{r^2}{a}}\left(-\frac{2r^\mu}{a} + \frac{\partial}{\partial r^\mu}\right)\left(-\frac{2r^\nu}{a} + \frac{\partial}{\partial r^\nu}\right)$$

$$\left[\frac{\partial}{\partial r^\mu}, r^\nu\right] = g^{\mu\nu}$$

For any graph one ,obtains

$$G = G(m_q, p, q_1, q_2, \Lambda) = \int_0^\infty d^3\alpha\, F(\alpha_1, \alpha_2, \alpha_3, m_q, p, q_1, q_2, \Lambda)$$

The set of Schwinger parameters α_i *can be turned into a simplex by introducing an additional t–integration via the identity*

$$1 = \int_0^\infty dt\, \delta\left(t - \sum_{i=1}^{3} \alpha_i\right)$$

$$G = \int_0^\infty d^3\alpha \int_0^\infty dt\, \delta\left(t - \sum_{i=1}^{3} \alpha_i\right) F(\alpha_1, \alpha_2, \alpha_3, \cdots) = \{\alpha_i = t\alpha_i\} =$$

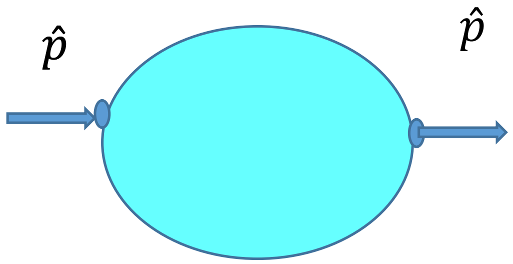
$$= \int_0^\infty dt\, t^2 \int_0^1 d^3\alpha \delta\left(1 - \sum_{i=1}^{3} \alpha_i\right) F(t\alpha_1, t\alpha_2, t\alpha_3, \cdots)$$

One can remove all possible thresholds present in the initial quark diagram by cutting the scale integration at the upper limit corresponding to the introduction of an infrared cutoff

$$\int_0^\infty dt \rightarrow \int_0^{\frac{1}{\lambda^2}} dt$$

So

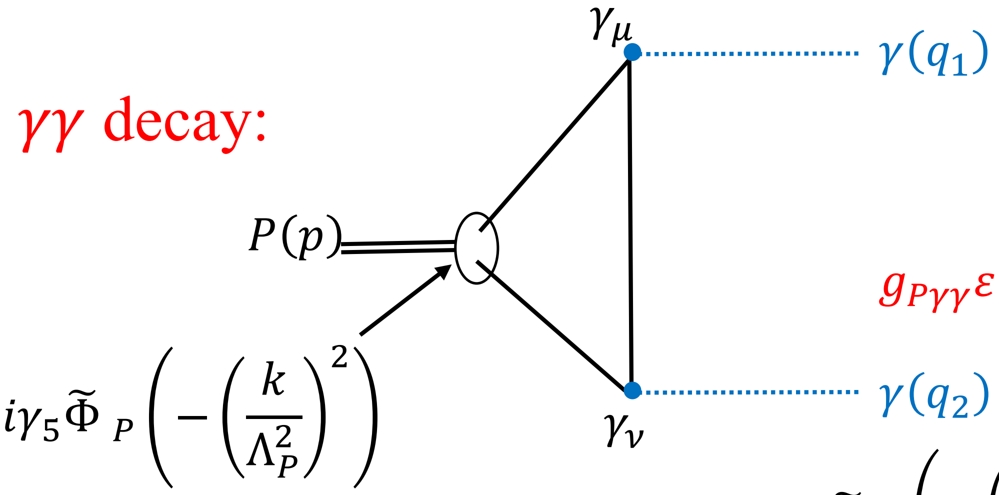
$$G^c = \int_0^{\frac{1}{\lambda^2}} dt\, t^{n-1} \int_0^1 d^n\alpha\, \delta\left(1 - \sum_{i=1}^n \alpha_i\right) F(t\alpha_1, t\alpha_2, \cdots, t\alpha_n)$$



$$\tilde{\Pi}'_M(p^2) = F_{PP}(p^2, m_q, \Lambda_M)$$

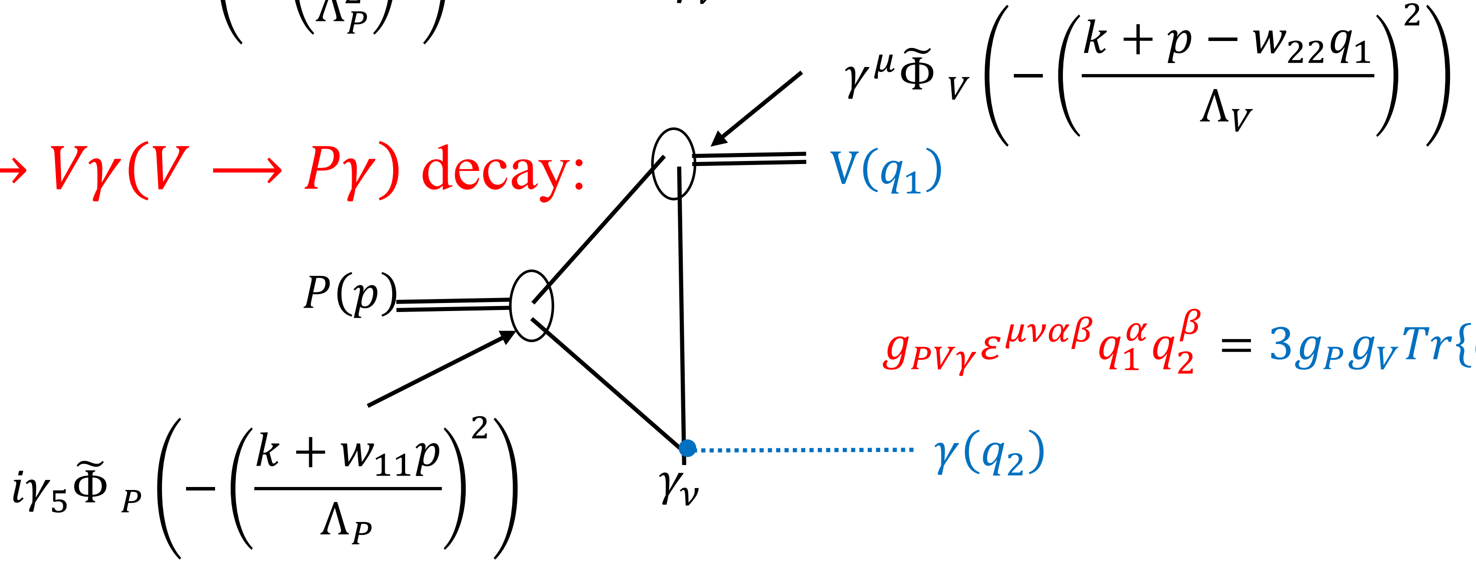
$$\tilde{\Pi}'_V(p^2) = F_{VV}(p^2, m_q, \Lambda_M)$$

$P \rightarrow \gamma\gamma$ decay:



$$g_{P\gamma\gamma}\varepsilon^{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta = 3g_P Tr\{\lambda_P Q^2\}F_{P\gamma\gamma}(p^2, q_1^2, q_2^2, m_q, \Lambda_P)$$

$P \rightarrow V\gamma(V \longrightarrow P\gamma)$ decay:



$$g_{PV\gamma}\varepsilon^{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta = 3g_P g_V Tr\{Q\{\lambda_V, \lambda_P\}\lambda_P\}F_{PV\gamma}(p^2, q_1^2, q_2^2, m_q, \Lambda_P, \Lambda_V)$$

Model parameters:

Universal cutoff parameter λ:

$\lambda = 0{,}181\ GeV$

Constituent quark masses

$m_u = m_d = 0{,}241\ GeV$

$m_s = 0{,}428\ GeV$

Size parameters of hadrons Λ_M

$\Lambda_{\rho,\omega,\phi} = 0{,}295 GeV,$

$\boldsymbol{\Lambda_{\eta,\eta'}}$ – ***have to be determined***

Main Properties of η, η' Mesons

$M_\eta = 547.51 \pm 0.18\ MeV$		$M_{\eta'} = 957.78 \pm 0.14\ MeV$	
$\Gamma_\eta = 1.30 \pm 0.07\ KeV$		$\Gamma_{\eta'} = 0.203 \pm 0.016\ MeV$	
$\eta \to \gamma\gamma$	39%	$\eta' \to \pi^+\pi^-\eta$	44%
$\eta \to \pi^0\pi^0\pi^0$	32%	$\eta' \to \rho^0\gamma$	29%
$\eta \to \pi^+\pi^-\pi^0$	23%	$\eta' \to \pi^0\pi^0\eta$	21%
$\eta \to \pi^+\pi^-\gamma$	5%	$\eta' \to \omega\gamma$	3%
		$\eta' \to \gamma\gamma$	2%

In order to quantify the mixing in the η-η' systems, one have to define the mixing scheme.

1. $\bar{q}q - \bar{s}s$ mixing

In this scheme η-η' mixing is defined as

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = U(\varphi) \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

where

$$U(\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

The flavor structure of η and η' :

$$\eta_q \Rightarrow \frac{1}{\sqrt{2}}\left(\bar{u}u + \bar{d}d\right)$$

$$\eta_s \Rightarrow \bar{s}s$$

2. *Mixing in the octet-singlet basis*

The pseudoscalar nonet can be displayed as:

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} \end{pmatrix}$$

So:

$$\eta = \eta_8 \cos\theta_\eta - \eta_0 \sin\theta_\eta$$
$$\eta' = \eta_8 \sin\theta_{\eta'} + \eta_0 \cos\theta_{\eta'}$$

The coupling constants for $\eta(\eta')$-quark interaction

1.$\bar{q}q - \bar{s}s$ *mixing*

$$h_\eta(\varphi, \Lambda_\eta) = \frac{2}{F_{PP}(m_\eta^2, m_n, \Lambda_\eta)\cos^2\varphi + F_{PP}(m_\eta^2, m_s, \Lambda_\eta)\sin^2\varphi}$$

$$h_{\eta'}(\varphi, \Lambda_{\eta'}) = \frac{2}{F_{PP}(m_{\eta'}^2, m_s, \Lambda_{\eta'})\cos^2\varphi + F_{PP}(m_{\eta'}^2, m_n, \Lambda_{\eta'})\sin^2\varphi}$$

2.Mixing in the octet- singlet basis

$$h_{\eta_0}(x, y) = \frac{2}{\frac{1}{3}(2F_{PP}(x, m_n, y) + F_{PP}(x, m_s, y))}$$

$$h_{\eta_8}(x, y) = \frac{2}{\frac{1}{3}(F_{PP}(x, m_n, y) + 2F_{PP}(x, m_s, y))}$$

$m_n = m_u = m_d = 0{,}241\ GeV$
$m_s = 0{,}428\ GeV$

To determine the values of mixing angles and parameters $\Lambda_{\eta,\eta'}$ we use experimental data on the constants of radiative decays of η and η' mesons and decays $\rho \longrightarrow \eta\gamma\lambda, \eta' \longrightarrow \rho\gamma$

$P \to \gamma\gamma$ decay:

Decay amplitude:

$$A(P \to \gamma\gamma) = e^2 g_{P\gamma\gamma} \varepsilon_{\mu\nu\alpha\beta} \epsilon^{\mu} \epsilon^{\nu} q_1^{\alpha} q_2^{\beta}$$

Decay width:

$$W(P \to \gamma\gamma) = \frac{\pi}{4} \alpha^2 m_P^3 g_{P\gamma\gamma}^2$$

$$W_{\text{экс}}(\eta \to \gamma\gamma) = (0{,}46 \pm 0{,}04)\ \text{KeV},$$

$$W_{\text{экс}}(\eta' \to \gamma\gamma) = (4{,}27 \pm 0{,}19)\ \text{KeV}.$$

$$g_{\eta\gamma\gamma}^{\exp} = 0{,}259\ \text{GeV}^{-1}, \qquad g_{\eta'\gamma\gamma}^{\exp} = 0{,}341\ \text{GeV}^{-1}$$

$\bar{q}q - \bar{s}s$ mixing

$$g_{\eta\gamma\gamma}(\varphi, \Lambda_\eta) = \frac{\sqrt{3h_\eta(\varphi, \Lambda_\eta)}}{\pi}\left(\frac{1}{\sqrt{2}}\frac{5}{9}F_{P\gamma\gamma}(m_\eta^2, 0,0, m_n, \Lambda_\eta)\, cos\varphi - \frac{1}{9}F_{P\gamma\gamma}(m_\eta^2, 0,0, m_s, \Lambda_\eta)\, sin\varphi\right)$$

$$g_{\eta'\gamma\gamma}(\varphi, \Lambda_{\eta'}) = \frac{\sqrt{3h_{\eta'}(\varphi, \Lambda_{\eta'})}}{9\pi}\left(\frac{5}{\sqrt{2}}F_{P\gamma\gamma}\left(m_{\eta'}^2, 0,0, m_n, \Lambda_{\eta'}\right)\, sin\varphi + F_{PVV}(m_{\eta'}^2, 0,0, m_s, \Lambda_{\eta'})\, cos\varphi\right)$$

Mixing in the octet-singlet basis

$$g_{\eta\gamma\gamma}(\theta_\eta, \Lambda_{\eta_0}, \Lambda_{\eta_8}) = \frac{\sqrt{3}}{\pi}\left(g_{\eta 8\gamma\gamma}(m_\eta^2, \Lambda_{\eta_8})\, cos\theta_\eta - g_{\eta 0\gamma\gamma}(m_\eta^2, \Lambda_{\eta_0})\, sin\theta_\eta\right),$$

$$g_{\eta'\gamma\gamma}(\theta_{\eta'} \Lambda_{\eta_0}, \Lambda_{\eta_8}) = \frac{\sqrt{3}}{\pi}\left(g_{\eta 8\gamma\gamma}(m_{\eta'}^2, \Lambda_{\eta_8})\, sin\theta_{\eta'} + g_{\eta 0\gamma\gamma}(m_{\eta'}^2, \Lambda_{\eta_0})\, cos\theta_{\eta'}\right),$$

$$g_{\eta 0\gamma\gamma}(x, y) = \sqrt{h_{\eta 0}(x, y)}\frac{1}{\sqrt{3}}\left(\frac{5}{9}F_{P\gamma\gamma}(x, 0{,}0, m_n, y) + \frac{1}{9}F_{P\gamma\gamma}(x, 0{,}0, m_s, y)\right),$$

$$g_{\eta 8\gamma\gamma}(x) = \sqrt{h_{\eta 8}(x, y)}\frac{1}{\sqrt{6}}\left(\frac{5}{9}F_{P\gamma\gamma}(x, 0{,}0, m_n, y) - \frac{2}{9}F_{P\gamma\gamma}(x, 0{,}0, m_s, y)\right).$$

$\rho \to \eta\gamma$ and $\eta' \longrightarrow \rho\gamma$ decays:

Decay amplitude:

$$A(V \to P\gamma) = e g_{VP\gamma} \varepsilon^{\mu\nu\alpha\beta} \epsilon^{\mu}(q_{\gamma}) \epsilon^{\nu}(p_V) q_{\gamma}^{\alpha} p_V^{\beta}$$

$$A(P \to V\gamma) = e g_{PV\gamma} \varepsilon^{\mu\nu\alpha\beta} \epsilon^{\mu}(q_{\gamma}) \epsilon^{\nu}(p_V) q_{\gamma}^{\alpha} p_V^{\beta}$$

Decay width:

$$W(V \to P\gamma) = \frac{\alpha}{24} m_V^3 \left(1 - \frac{m_P^2}{m_V^2}\right) g_{VP\gamma}^2,$$

$$W(P \to V\gamma) = \frac{\alpha}{8} m_P^3 \left(1 - \frac{m_V^2}{m_P^2}\right) g_{PV\gamma}^2.$$

$$g_{\rho\eta\gamma}^{\exp} = 1{,}47^{+0{,}25}_{-0{,}28}\ \mathrm{GeV}^{-1}, \qquad g_{\eta'\rho\gamma}^{\exp} = 1{,}31 \pm 0{,}06\,\mathrm{GeV}^{-1}$$

$\bar{q}q - \bar{s}s$ mixing

The best agreement with experimental data can be achieved with

$$\varphi = 39{,}3°$$

$$\Lambda_\eta = 0{,}82 GeV;\ \Lambda_{\eta'} = 0{,}38 GeV$$

Mixing in the octet-singlet basis

The best agreement with experimental data can be achieved with

$$\theta_\eta = -15{,}4°,$$

$$\theta_{\eta'} = 58° \text{ or } \theta_{\eta'} = -17{,}9°.$$

$$\Lambda_{\eta_0} = 0{,}75 GeV;\ \Lambda_{\eta_8} = 0{,}28 GeV$$

NUMERICAL RESULTS FOR RADIATIVE DECAYS

$V \to P\gamma$

$g_{V\eta\gamma}$(ГэВ$^{-1}$)	Experiment	$q\bar{q} - s\bar{s}$ -mixing $\varphi = 39{,}3°$	OS-mixing $\theta_\eta = -15{,}4°$
$g_{\rho\eta\gamma}$	$1{,}47^{+0{,}25}_{-0{,}28}$	1,48	1,495
$g_{\omega\eta\gamma}$	$0{,}53 \pm 0{,}4$	0,49	0,49
$g_{\varphi\eta\gamma}$	$0{,}69 \pm 0{,}2$	0,715	0,72

One can see the satisfactory agreement with experimental data

NUMERICAL RESULTS FOR RADIATIVE DECAYS

$P \to V\gamma$

$g_{\eta' V\gamma}$ (ГэВ$^{-1}$)	Experiment	$q\bar{q} - s\bar{s}$ **-mixing** $\varphi = 39{,}3°$	**OS-mixing**	
			$\theta_{\eta'} = 58°$	$\theta_{\eta'} = -17{,}9°$
$g_{\eta'\rho\gamma}$	$1{,}31 \pm 0{,}06$	1,065	1,55	1,05
$g_{\eta'\omega\gamma}$	$0{,}45 \pm 0{,}03$	0,353	0,514	0,330
$g_{\eta'\varphi\gamma}$	$1{,}00^{+0{,}28}_{-0{,}21}$	0,761	0,381	0,786

analysis of the results indicates that the $\theta_{\eta'} = -17{,}9°$ *is preferable*