Bremsstrahlung of single gauge boson production on high energy electron-photon collisions

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The problem of the bremsstrahlung contribution calculation as a part of the radiative corrections in the case of single gauge boson production processes was discussed. It was shown that hard photon bremsstrahlung contribution can be divided into the divergent and finite terms. The exact calculation of soft photon bremsstrahlung and infrared part of hard photon bremsstrahlung was presented in frame of the dimensional regularization scheme.

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I. INTRODUCTION

The calculation of radiative corrections for the processes of elementary particles interactions is an important and time-consuming problem of quantum field theory. The presence of unphysical infrared (IR) and ultraviolet (UV) divergences requires the usage of specific methods for their parameterization and cancellation. In general, the presence of divergences imposes significant restrictions on the possibility of direct numerical analysis of the observed quantities, including the total and differential cross sections of the processes under consideration. In this regard, the analytical covariant description of the processes is essential.

Conventionally, the contribution of the lowest-order radiative corrections can be divided into two categories: the contribution of virtual particles, or one-loop corrections (V-terms) and the contribution of real photons, or bremsstrahlung (R-terms). UV-divergences are eliminated by taking into account the contribution of counterterms in accordance with the chosen renormalization scheme. IR-divergences are eliminated by taking into account the R-contribution.

Problems associated with the V-contribution are substantially resolved. Taking into account the contribution of real photons is a much more complicated task, requiring an exceptional approach to each individual process and its kinematics. Due to this, the problem of accounting for the R-contribution requires a larger number of unique analytical results. It should be noted that Belarusian scientists also played an important role in developing effective methods for covariant analytical calculations of the R-contribution for various processes [1–3].

In this paper, results concerning a number of problems that arise in calculating the R-contribution for various processes was obtained. In accordance with the common methods, the contribution of soft photons is calculated using the dimensional regularization method, as well as calculation of the contribution of hard bremsstrahlung with the separation of

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the IR diverging part for processes based on electron-photon collisions, which will take place at the International Linear Collider (ILC) [4, 5] and are important for the study of the Standard Model deviations [6–11].

II. SOFT PHOTON BREMSSTRAHLUNG

The simplest way to taking into account the bremsstrahlung contribution for elimination of IR divergences is the soft-photon approximation. If the final photons have sufficiently low energy, in the propagators of the processes amplitude 4-momentum of photon can be neglected and the matrix element square for the bremsstrahlung process can be represented as follows:

$$d\sigma_{\mathrm{B}} = \delta_{\mathrm{R}}^{\mathrm{SB}} \cdot d\sigma_{\mathrm{B}}. \tag{1}$$

The factor $\delta_{\mathrm{R}}^{\mathrm{SB}}$ can be written as

$$\delta_{\mathrm{R}}^{\mathrm{SB}} = -\frac{\alpha}{2\pi^2}\left(\sum_{i}^{m} I_i + \sum_{i>j}^{m} A_{ij}^{C} A_{ij}^{IF} I_{ij}\right), \tag{2}$$

where A_{ij}^{C}, A_{ij}^{IF} – charge asymmetry and state asymmetry coefficients, respectively. They are equal to $+1$ if particles with momenta p_i and p_j have the same charges / both are finite (initial) particles and -1 otherwise. Summation is performed over the momenta of external charged particles.

The functions I_i and I_{ij} are expressed as follows:

$$I(p_i) = m_i^2 I_i, \qquad I(p_i, p_j) = x_{ij} I_{ij}, \tag{3}$$

where $x_{ij} = 2p_i \cdot p_j$ and functions $I(p_i)$ and $I(p_i, p_j)$ are the following integrals

$$I(p_i, p_j) = \int_0^{\Delta E} \frac{d^3 q}{(2\pi)^3 2\, q^0} \frac{1}{(p_i q)(p_j q)}, \qquad I(p_i) = I(p_i, p_i). \tag{4}$$

To calculate these integrals, one make the following change of variables

$$p = \zeta p_i, \qquad k = p_j \tag{5}$$

such as $(p-k)^2 = 0$, and the vector $p - k$ is isotropic. In this case, the initial integral can be rewritten in the form:

$$I(p_i, p_j) = \zeta \int_0^{\Delta E} \frac{d^3 q}{(2\pi)^3 2\, q^0} \frac{1}{(pq)(kq)}. \tag{6}$$

Performing analytical continuation to the n-dimensions and transforming integration to integration over angles, one can obtain [12]

$$I(p_i, p_j) = \frac{2}{(2\sqrt{\pi})^n \Gamma(n/2-1)} \int_0^1 dx \frac{1}{\mu^{n-4}} \int_0^{\Delta E} (q^0)^{n-5} dq^0 \int_0^{\pi} (\sin\theta)^{n-3} d\theta \frac{1}{\left[(u^0)^2 - |\vec{u}|^2 \cos\theta_x\right]^2} \tag{7}$$

Here x is Feynman parameter and $u = px + k(1-x)$. Integrating over the photon energy, one come to the following expression:

$$\frac{1}{\mu^{n-4}}\int_0^{\Delta E} dq^0 (q^0)^{n-5} = \frac{(\Delta E/\mu)^{n-4}}{n-4} = \frac{1}{n-4}\left[1 + (n-4)\ln\frac{\Delta E}{\mu} + ...\right]. \tag{8}$$

Soft photon emission is isotropic. Therefore, one can always choose a coordinate frame so that one of the axes coincides in direction with one of the vectors, and, accordingly, $\theta_\alpha = \theta$. Therefore, immediately obtain:

$$\int_0^{\pi} \frac{(\sin\theta)^{n-3} d\theta}{\left[(u^0)^2 - |\vec{u}|^2\cos\theta\right]^2} = \int_{-1}^{1} \frac{((1-\xi^2)^{n/2-2} d\xi}{\left[(u^0)^2 - |\vec{u}|^2\xi\right]^2} = \tag{9}$$
$$= \int_{-1}^{1} \frac{d\xi}{\left[(u^0)^2 - |\vec{u}|^2\xi\right]^2}\left[1 + \frac{1}{2}(n-4)\ln(1-\xi^2) + ...\right]$$

Comparing the last two expressions and discarding all terms except linear on $1/(n-4)$

$$\frac{1}{\mu^{n-4}}\int_0^{\Delta E} dq^0 (q^0)^{n-5} \int_0^{\pi} \frac{(\sin\theta)^{n-3} d\theta}{\left[(u^0)^2 - |\vec{u}|^2\cos\theta\right]^2} = \tag{10}$$
$$= \int_{-1}^{1} \frac{d\xi}{\left[(u^0)^2 - |\vec{u}|^2\xi\right]^2}\left[\frac{1}{n-4} + \ln\frac{\Delta E}{\mu} + \frac{1}{2}\ln(1-\xi^2)\right] =$$
$$= 2\left[\frac{1}{n-4} + \ln\frac{2\Delta E}{\mu}\right]/u^2 + \frac{u^0}{|\vec{u}|\cdot u^2}\ln\frac{u^0 - |\vec{u}|}{u^0 + |\vec{u}|}.$$

Expanding n near 4 gives

$$I(p_i, p_j) = \zeta\frac{1}{2(2\pi)^2}\left(R_1 + R_2\right), \tag{11}$$

$$R_1 = \int_0^1 \frac{dx}{u^2}\left[-\Delta^{\mathbf{IR}} + \ln\frac{4\Delta E^2}{\mu^2}\right], \tag{12}$$

$$R_2 = \int_0^1 \frac{dx}{u^2}\frac{u^0}{|\vec{u}|}\ln\frac{u^0 - |\vec{u}|}{u^0 + |\vec{u}|}, \tag{13}$$

Here $\Delta^{\mathbf{IR}} \equiv \frac{1}{\epsilon} = \frac{2}{4-n} - \gamma_E + \ln 4\pi$ is IR-regulator in the $\overline{\text{MS}}$-scheme. Using the obvious expression

$$u^2 = k^2 + 2k\cdot(p-k) = k^2 + 2vl, \tag{14}$$
$$l = p^0 - k^0, \qquad v = \frac{2k\cdot(p-k)}{l} = \frac{p^2-k^2}{2l},$$

is easy to obtain an expression for R_1:

$$R_1 = \int_0^1 \frac{dx}{k^2 + 2vl}\left[-\Delta^{\mathbf{IR}} + \ln\frac{4\Delta E^2}{\mu^2}\right] = \frac{1}{p^2-k^2}\ln\frac{p^2}{k^2}\left[-\Delta^{\mathbf{IR}} + \ln\frac{4\Delta E^2}{\mu^2}\right] \tag{15}$$

Integral R_2 is identical to the integral calculated by 't Hooft and Veltman [13] and is equal to

$$R_2 = \frac{2}{p^2-k^2}\left[\frac{1}{4}\ln^2\frac{u^0 - |\vec{u}|}{u^0 + |\vec{u}|} + \mathrm{Li}_2\left(1 + \frac{u^0 + |\vec{u}|}{v}\right) + \mathrm{Li}_2\left(1 + \frac{u^0 - |\vec{u}|}{v}\right)\right]_{u=k}^{u=p} \tag{16}$$

Thus, the functions I_i I_{ij} will be expressed as follows:

$$I_i = \pi \left[-\Delta^{\mathbf{IR}} + \log \frac{4\Delta E^2}{\mu^2} + \frac{p^0}{|\mathbf{p}|} \log \frac{p^0 - |\mathbf{p}|}{p^0 + |\mathbf{p}|} \right] \tag{17}$$

$$I_{ij} = 2\pi \frac{\zeta(x_{ij})}{\zeta^2 m_i^2 - m_j^2} \left[\frac{1}{2} \log \frac{\zeta^2 m_i^2}{m_j^2} \left(-\Delta^{\mathbf{IR}} + \log \frac{4\Delta E^2}{\mu^2} \right) + \right. \tag{18}$$

$$\left. + \left\langle \frac{1}{4} \log^2 \frac{u^0 - |\mathbf{u}|}{u^0 + |\mathbf{u}|} + \mathrm{Li}_2 \left(1 - \frac{u^0 + |\mathbf{u}|}{v} \right) + \mathrm{Li}_2 \left(1 - \frac{u^0 - |\mathbf{u}|}{v} \right) \right\rangle_{u=p_j}^{u=\zeta p_i} \right] \tag{19}$$

with

$$\zeta = \frac{x_{ij} + \sqrt{x_{ij}^2 - 4m_i^2 m_j^2}}{2m_i^2}, \qquad v = \frac{\zeta^2 m_i^2 - m_j^2}{2(\zeta p_i^0 - p_j^0)}, \qquad x_{ij} = 2p_i \cdot p_j. \tag{20}$$

III. HARD PHOTON BREMSSTRAHLUNG

The soft photon bremsstrahlung contribution depends on the collider energy resolution. However, in the soft photon approximation this energy should be much less than the interaction energy (including the masses of particles). To avoid this dependence one can take into account the hard photon bremsstrahlung contribution. Since the kinematics of the bremsstrahlung process is different from the kinematics of the initial process, it is impossible to fully algorithmize the calculation of the contribution of the bremsstrahlung. However, note the following important feature: the tensor structure of the matrix elements of the bremsstrahlung processes allows naturally to separate the IR-divergent and finite parts of the matrix elements square of the processes under consideration:

$$|\mathcal{M}_{\mathrm{R}}|^2 = |\mathcal{M}_{\mathrm{R}}^{\mathrm{F}}|^2 + |\mathcal{M}_{\mathrm{R}}^{\mathrm{IR}}|^2, \tag{21}$$

and IR-divergent term can be factorized with matrix element in Born approximation $|\mathcal{M}_{\mathrm{R}}^{\mathrm{IR}}|^2 \propto |\mathcal{M}_{\mathrm{B}}$. Respectively, the differential cross section of bremsstrahlung process can be also factorized:

$$d\sigma_{\mathrm{R}} = \delta_{\mathrm{R}}^{\mathrm{IR}} \cdot d\sigma_{\mathrm{B}}. \tag{22}$$

Next, consider the special case of the single gauge boson production process with the additional bremsstrahlung photon:

$$e^-(p, m_e) + \gamma(k, 0) \to C^-(p_1, m_c) + N^0(k_1, m_n) + \gamma(q, 0).$$

To describe the kinematics of the initial process (without a finite photon), two parameters are needed, for which Mandelstam invariants are often used in covariant calculations:

$$s = (p + k)^2, \qquad t_1 \equiv -Q^2 = (k - k_1)^2.$$

For describing kinematic of bremsstrahlung process needs three more invariants

$$s_1 = (p_1 + q)^2; \qquad s_2 = (p_1 + k_1)^2; \qquad t_2 = (p - q)^2.$$

The expression of the total cross section for the such type of processes has the following form:

$$\sigma_{\mathrm{R}} = \frac{(2\pi)^{-4}}{4(s - m_e^2)^2} \int |\mathcal{M}_{\mathrm{R}}|^2 \, \frac{dt_1 ds_1 ds_2 dt_2}{8\sqrt{-\Delta_4}}. \tag{23}$$

Comparing with the (22), gets that $\delta_{\mathrm{R}}^{\mathrm{IR}}$ depends on all new invariant and can be expressed as follows:

$$\delta_{\mathrm{R}}^{\mathrm{IR}} = -\frac{\alpha}{\pi^2}\int\left(\frac{m_e^2}{(m_e^2-t_2)^2}+\frac{m_c^2}{(s_1-m_c^2)^2}-\frac{Q^2+m_e^2+m_c^2}{(m_e^2-t_2)(s_1-m_c^2)}\right)\frac{ds_1dt_2ds_2}{\sqrt{-\Delta_4}} = \tag{24}$$

$$= -\frac{\alpha}{\pi^2}(m_e^2\delta_1+m_1^2\delta_2+(m_1^2+m_e^2-t_1)\delta_3), \tag{25}$$

where Δ_4 is the Gram determinant. All kinematic boundaries can be obtained from the condition $\Delta_4 = 0$. Gram determinant can be presented in the following form $\Delta_4 = -\lambda(s_2^+ - s_2)(s_2^- - s_2)$, $\lambda \equiv \lambda(s_1, t_1, m_e^2)$. It is easy to verify that the factor $\delta_{\mathrm{R}}^{\mathrm{IR}}$ coincides with $\delta_{\mathrm{R}}^{\mathrm{SB}}$, but is expressed using invariants of different kinematics.

For another invariants one have following kinematical restrictions for IR-parametrization in the laboratory system with $\vec{p} = 0$:

$$t_2^{\pm} = -s_1+t_1+\frac{1}{2s_1}\left[(s_1+m_1^2)(s_1+m_1^2-t_1)\pm\sqrt{\lambda(s_1,m_1^2,t_1)\lambda(s_1,m_1^2,0)}\right], \tag{26}$$

$$\bar{s}_1 = \frac{(s+t_1-m_e^2-M^2)\,(m_e^2M^2-st_1^2)}{(s-m_e^2)\,(M^2-t_1)}, \tag{27}$$

$$\underline{s}_1 = m_1^2+F, \tag{28}$$

$$t_1 = M_1^2-\frac{1}{2s}\left[(s-m_e^2)(s-m_1^2+M_1^2)\mp(s-m_e^2)\sqrt{\lambda(s,m_1^2,M_1^2)}\right]. \tag{29}$$

where $F = 2m_1\Delta E$. Integration over s_2 is trivial. Calculate several integrals by t_2:

$$\int_{t_2^-}^{t_2^+}\frac{dt_2}{(t_2-m_e^2)^2} = \frac{\sqrt{\lambda}}{m_e^2(s_1-m_1^2)}, \tag{30}$$

$$\int_{t_2^-}^{t_2^+}dt_2 = \sqrt{\lambda}\frac{(s_1-m_1^2)}{s_1}, \tag{31}$$

$$\int_{t_2^-}^{t_2^+}\frac{dt_2}{t_2-m_e^2} = \ln\left[\frac{1-\beta}{1+\beta}\right]. \tag{32}$$

where $\beta = \sqrt{\lambda}/(s_1+m_e^2-t_1)$. Using this results and additionally preforming integration over s_2 gives following result for δ_1 and δ_2 in limit $\Delta E \to 0$:

$$\delta_1 = -\frac{1}{m_e^2}\ln\left[\frac{2\Delta E m_1}{\bar{s}_1-m_1^2}\right], \tag{33}$$

$$\delta_2 = -\frac{1}{m_1^2}\left\{\ln\left[\frac{2\Delta E m_1}{\bar{s}_1-m_1^2}\right]+\ln\left[\frac{\bar{s}_1}{m_1^2}\right]\right\}. \tag{34}$$

Last form factor δ_3 have following form:

$$\delta_3 = \int_{m_1^2+F}^{\bar{s}_1}\frac{\ln\left[\frac{1-\beta}{1+\beta}\right]}{\sqrt{\lambda}(s_1-m_1^2)}ds_1. \tag{35}$$

This integral is not calculated by conventional methods, so calculate it approximately. To do this, one must divide the finite and divergent parts of δ_3:

$$\delta_3 = \delta_3^{\mathrm{IR}}+\delta_3^{\mathrm{F}}. \tag{36}$$

The divergent part must be determined as accurately as possible. Therefore, one use the expansion of the logarithm

$$\ln\left[\frac{1-\beta}{1+\beta}\right] = \sum_{i=0}^{\infty}\frac{\beta^{2i+1}}{2i+1} \tag{37}$$

and integrate it. Assembling the parts containing the divergences, it is easy to see that this is the expansion of the following function:

$$\delta_3^{\rm IR} = \frac{1}{\sqrt{\lambda_t}}\ln\left[x_t\right]\ln\left[\frac{2\Delta E m_1}{\bar{s}_1 - m_1^2}\right]. \tag{38}$$

The notation is introduced here

$$\lambda_t = \lambda(t_1, m_e^2, m_1^2), \qquad \beta_t = \sqrt{\lambda_t}/(m_e^2 + m_1^2 - t_1), \qquad x_t = \frac{1-\beta_t}{1+\beta_t}. \tag{39}$$

In the ultrarelativistic approximation ($m_e \to 0$)

$$\delta_3 = \int_{m_1^2+F}^{\bar{s}_1} \frac{\ln\left[\frac{s_1}{m_e^2}\right] - \frac{1}{2}\ln\left[\frac{s_1-t_1}{s}\right]}{(s_1-t_1)(s_1-m_1^2)} ds_1. \tag{40}$$

Integrating and comparing the expression of form factors, obtain the final expression for $\delta_{\rm R}^{\rm IR}$ [14]

$$\begin{aligned}\delta_{\rm R}^{\rm IR} = &-\frac{\alpha}{2\pi}\left\langle \log\frac{4\Delta E^2 m_c^2}{(\bar{s}_1 - m_c^2)^2}\left[2 - \frac{1}{\beta_t}\log x_t\right] + \Re\left\{\log s_1 - 2\log\frac{m_c^2 - t}{s_1 - t} - \right.\right.\\ &- \log^2(s_1 - t) + \log(\bar{s}_1 - m_c^2)(2\log(s_1-t) - \log m_e^2 s_1) + \\ &\left.\left. + \log m_e^2 s_1 \log\frac{m_c^2(s_1-t)}{-t} - {\rm Li}_2\frac{s_1}{m_c^2} + {\rm Li}_2\frac{s_1}{t} + 2{\rm Li}_2\frac{s_1 - t}{m_c^2 - t}\right\}_{s_1=m_c^2}^{s_1=\bar{s}_1}\right\rangle\end{aligned} \tag{41}$$

IV. NUMERICAL ANALYSIS

In order to evaluate the contribution of the finite part of the R-contribution, it makes sense to conduct a numerical analysis for a number of processes, taking into account the above calculations. Comparing the results obtained with the calculations of other authors, one can approximately estimate the contribution of the finite part of the R-contribution and draw conclusions about the need to taking it into account. Differential radiative corrections for various processes of single gauge bosons production in high-energy electron-photon collisions are shown in Fig. 1. It show that, by virtue of the principle of gauge cancellation, the absolute value of the radiative corrections increases with increasing energy. Also notice that the W–boson production process demonstrates the highest value of corrections, which reach –35% for momentum transfer $|Q| = 170$ GeV and energy of colliding beams $\sqrt{s} = 1$ TeV.

As noted above, a numerical analysis can be successfully performed only at the level of the total cross sections of processes. For numerical calculations, the adaptive quasi Monte-Carlo method *Vegas* was chosen, the cutting angle was taken equal to $\Delta\vartheta = 20°$. The total relative radiative corrections for the same processes are presented in Fig. 2. At an interaction energy $\sqrt{s} = 1$ TeV, the radiative corrections for the processes of neutral

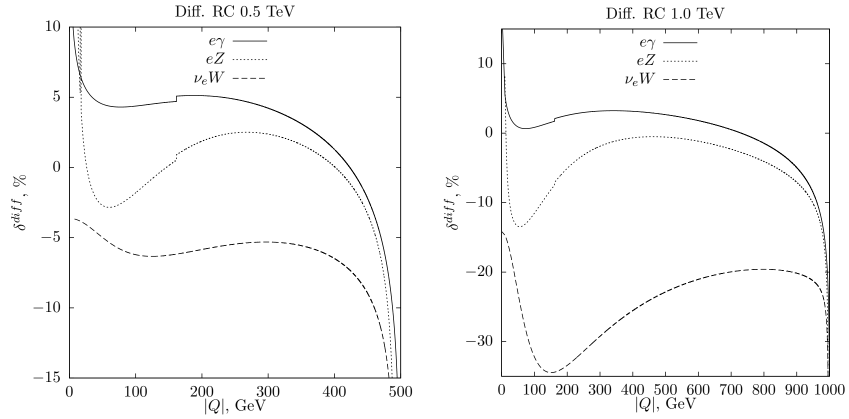

Figure 1: The differential radiative corrections for a set of processes.

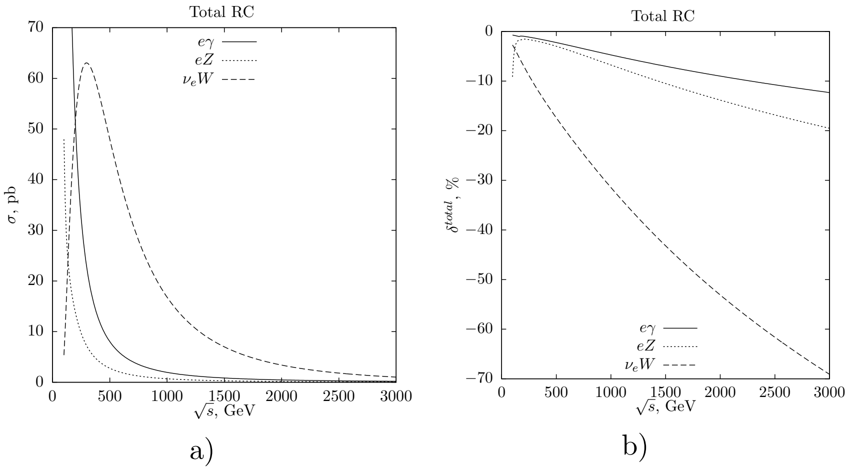

Figure 2: The total cross sections including the lowest order radiative corrections a) and relative radiative corrections b) for a set of processes.

gauge bosons production are of the order of 5% in absolute value, which, in general, is consists with the data of other researchers. However, for the $W-$boson production process at the same energy, the value of radiative corrections reaches 30%, which is significantly different from the data presented in other sources. Obviously an additional peak should be observed in the finite R-contribution, which will lead to an increase in the relative radiative correction for the process by a value of about 20%.

V. CONCLUSION

The lowest-order radiative corrections is important for elementary particle physics for increasing the theoretical accuracy of the calculation of processes. The usage of exact covariant methods in calculations of radiative corrections is extremely important both for confirming the predictions of the Standard Model and for searching physics beyond it.

The results obtained in this work can be applied in calculations of the R-contribution as a part of radiative corrections for processes in experiments of particles collisions at colliders of all types. The dimensional regularization method used allows accounting for bremsstrahlung in the most correct and modern way. Analytical accounting of the IR-divergent part of the contribution of hard bremsstrahlung is important for the numerical stability of the calculations of the R-contribution and allows covariance of the obtained results, as well as elimination of the dependence on the experimental parameters. The ultra-relativistic approximation, where the mass of the electron was neglected, works well at energies of the order of 100 MeV, therefore the obtained data can certainly be considered to be sufficiently accurate. Numerical analysis showed that taking into account the finite part of the R-contribution is required. In particular, for processes of the $e\gamma \to CN(\gamma)$ type, contribution to the relative radiative corrections is exceptionally positive, and at high interaction energies it can reach 10–20%.

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