Resonant conversions of Majorana neutrinos in three generations

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Minsk, 2020

Investigation of MM's of massive neutrinos, propagating in dense matter and in large magnetic fields, is of great importance for elementary particle physics, as well as for neutrino astrophysics and cosmology.

The examples of the intensive magnetic fields (MF's) are:

MF's of coupled sunspots in preflare period;
MF's at the surface of neutron stars's and magnetars;
MF's within the jets of collapsars, **and so on.**

In the present work, amongst the neutrino MM's, we will be interested in the dipole magnetic and anapole moments. In the framework of the minimally extended SM with right-handed neutrino singlets added, the diagonal magnetic moment of the neutrino appeared to be very small

$$\mu_{\nu_l \nu_l} = 10^{-19}\ \mu_B \left(\frac{m_{\nu_l}}{\mathrm{eV}}\right),$$

Some extensions to the SM predict substantially larger values for the neutrino DMM's. So, when we are going to discuss the observable effects for the neutrino in electro-magnetic field we should chose the SM extension predicting rather large the DMM value but which does not contradict to current experimental data. For a such SM ex-tension we shall use the left right symmetric model (LRM) that is based on the $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ gauge group.

The aim of the present work is to investigate the behavior of the neutrino flux in

- Note that investigation of the effects connecting with the neutrino DMM's is a way to define the neutrino nature (Dirac or Majorana). There are two fundamental differences between the Dirac and Majorana neutrinos: (i) Majorana neutrinos can have only a transition magnetic moment; (ii) the right-handed Majorana neutrinos are not sterile, but interact as right-handed Dirac antineutrinos.

- It is worth noting that most of papers studying the behavior ofthe neutrino beam in the magnetic field are limited by two-flavor approximation. Within the three-neutrino generations this problem was investigated in (O.M.Boyarkin, Int. J. Modern Phys. A **34** (2019) 1950227) for the Dirac neutrinos. The goal of the present work is to consider the behavior of the Majorana neutrinos in an intensive magnetic field within three neutrino generations. As an example of such a field we shall consider the Sun's magnetic fields. In so doing of special interest are the magnetic fields of the solar sunspots which will be the source of the solar flare (SF).

- The SF formation starts from pairing big sunspots of opposite polarity (coupled sunspots --- CS's). The magnetic flux $10^{24}\ \mathrm{Gauss \cdot cm^2}$ erupting from the solar interior accumulates within the sunspots giving rise to the stored magnetic field. In so doing, the magnetic field value for the CS's could be increased from 10^4 Gs up to 10^5 Gs and upwards.

The existence of electromagnetic MM's caused by the radiative corrections could be taken into account in terms of the effective Lagrangian

$$\mathcal{L}_{eff} = \frac{e}{2}\overline{\nu}_l(x)\Big[\mu_{ll'}\sigma^{\beta\lambda} + a_{ll'}(\partial^{\beta}\gamma^{\lambda} - \partial^{\lambda}\gamma^{\beta})\Big](1-\gamma_5)\nu_{l'}(x)F_{\lambda\beta}(x) + \text{conj.},$$

We shall reason that magnetic fields in which the neutrino beam travels have the nonpotential character and exhibit the geometrical phase $\Phi(z)$

$$B_x \pm iB_y = B_{\perp}e^{\pm i\Phi(z)},$$

which is defined by the simple expression

$$\Phi(z) = \frac{\alpha\pi}{L_{mf}}z.$$

Then in the flavor basis the evolution equation will look like

$$i\frac{d}{dz}\begin{pmatrix}\nu_{eL}\\ \nu_{\mu L}\\ \nu_{\tau L}\\ \overline{\nu}_{eR}\\ \overline{\nu}_{\mu R}\\ \overline{\nu}_{\tau R}\end{pmatrix} = \mathcal{H}^M \begin{pmatrix}\nu_{eL}\\ \nu_{\mu L}\\ \nu_{\tau L}\\ \overline{\nu}_{eR}\\ \overline{\nu}_{\mu R}\\ \overline{\nu}_{\tau R}\end{pmatrix}, \qquad (9)$$

where

$$\mathcal{H}^M = \mathcal{U}\begin{pmatrix}\Sigma & 0\\ 0 & \Sigma\end{pmatrix}\mathcal{U}^{-1} + \begin{pmatrix} I_L & \mathcal{M}^M\\ -\mathcal{M}^M & I_R\end{pmatrix}, \qquad (10)$$

$$\Sigma = \begin{pmatrix} E_1 & 0 & 0\\ 0 & E_2 & 0\\ 0 & 0 & E_3\end{pmatrix}, \qquad I_L = \begin{pmatrix} V'_{eL} + \mathcal{A}_{\nu_L\nu_L} & 0 & 0\\ 0 & V_{\mu L} + \mathcal{A}_{\nu_L\nu_L} & 0\\ 0 & 0 & V_{\tau L} + \mathcal{A}_{\nu_L\nu_L}\end{pmatrix},$$

$$I_R = \begin{pmatrix} -V'_{eL} - \mathcal{A}_{\nu_R\nu_R} & 0 & 0\\ 0 & -V_{\mu L} - \mathcal{A}_{\nu_R\nu_R} & 0\\ 0 & 0 & -V_{\tau L} - \mathcal{A}_{\nu_R\nu_R}\end{pmatrix},$$

$$\mathcal{M}^M = \begin{pmatrix} 0 & \mu_{e\mu}B_\perp & -\mu_{e\tau}B_\perp \\ -\mu_{e\mu}B_\perp & 0 & \mu_{\mu\tau}B_\perp \\ \mu_{e\tau}B_\perp & -\mu_{\mu\tau}B_\perp & 0 \end{pmatrix}, \qquad \mathcal{U} = \begin{pmatrix} \mathcal{D} & 0 \\ 0 & \mathcal{D} \end{pmatrix},$$

$$\mathcal{D} = \exp(i\lambda_7\psi)\exp(i\lambda_5\phi)\exp(i\lambda_2\omega) = \begin{pmatrix} c_\omega c_\phi & s_\omega c_\phi & s_\phi \\ -s_\omega c_\psi - c_\omega s_\psi s_\phi & c_\omega c_\psi - s_\omega s_\psi s_\phi & s_\psi c_\phi \\ s_\omega s_\psi - c_\omega c_\psi s_\phi & -c_\omega s_\psi - s_\omega c_\psi s_\phi & c_\psi c_\phi \end{pmatrix},$$

$\psi = \theta_{23}$, $\phi = \theta_{13}$, $\omega = \theta_{12}$, $s_\psi = \sin\psi, c_\psi = \cos\psi$, and so on, the λ's are Gell-Mann matrices corresponding to the spin-one matrices of the $SO(3)$ group, V'_{eL} $(V_{\mu L}, V_{\tau L})$ is a matter potential describing interaction of the ν_{eL} $(\nu_{\mu L}, \nu_{\tau L})$ neutrinos with a dense matter,

$$V'_{eL} = V_{eL} + V^{\bar{\delta}}_{ee}, \qquad V_{eL} = \sqrt{2}G_F(n_e - n_n/2), \qquad V_{\mu L} = V_{\tau L} = -\sqrt{2}G_F n_n/2,$$

$$\mathcal{A}_{\nu_L\nu_L} = 4\pi a_{\nu_L\nu_L} j_z - \dot{\Phi}/2, \qquad \mathcal{A}_{\nu_R\nu_R} = 4\pi a_{\nu_R\nu_R} j_z - \dot{\Phi}/2,$$

and n_n is a neutron density.

Of course, to get the survival probabilities of definite flavor neutrinos, we can turn to the Hamiltonian (10) and find all possible resonant transitions in a flavor basis immediately. However, even though we work with the three component neutrino wave function $\Psi^T = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})$ the physical implications will be far from transparent [15]. For this reason we shall rotate the flavor basis in such a way to make the physical meanings more obvious. In so doing, when the mixing angles ψ and ϕ tend to zero, our results must convert into those obtained within the two-flavor approximation. This could be arranged by the following transformation

$$\begin{pmatrix} \nu_1'^M \\ \nu_2'^M \\ \nu_3'^M \\ \overline{\nu}_1'^M \\ \overline{\nu}_2'^M \\ \overline{\nu}_3'^M \end{pmatrix} = \mathcal{U}' \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \overline{\nu}_{eR} \\ \overline{\nu}_{\mu R} \\ \overline{\nu}_{\tau R} \end{pmatrix}, \tag{11}$$

$$\mathcal{U}' = \begin{pmatrix} \mathcal{D}' & 0 \\ 0 & \mathcal{D}' \end{pmatrix}, \qquad \mathcal{D}' = \exp(-i\lambda_5\phi)\exp(-i\lambda_7\psi) = \begin{pmatrix} c_\phi & 0 & s_\phi \\ -s_\phi s_\psi & c_\psi & c_\phi s_\psi \\ -s_\phi c_\psi & -s_\phi & c_\phi c_\psi \end{pmatrix}.$$

In this basis the Hamiltonian $\mathcal{H}'^M$ has the form

$$\mathcal{H}'^M = \begin{pmatrix} \mathcal{B}_v + \Lambda & \mathcal{M} \\ -\mathcal{M} & \mathcal{B}_v + \overline{\Lambda} \end{pmatrix}, \tag{12}$$

where

$$\mathcal{B}_v = \begin{pmatrix} -\delta^{12}c_{2\omega} & \delta^{12}s_{2\omega} & 0 \\ \delta^{12}s_{2\omega} & \delta^{12}c_{2\omega} & 0 \\ 0 & 0 & \delta^{31}+\delta^{32} \end{pmatrix}, \qquad \Lambda = \begin{pmatrix} V_{eL}^{eff}c_\phi^2 & 0 & V_{eL}^{eff}s_{2\phi}/2 \\ 0 & 0 & 0 \\ V_{eL}^{eff}s_{2\phi}/2 & 0 & V_{eL}^{eff}s_\phi^2 \end{pmatrix},$$

$$\overline{\Lambda} = \begin{pmatrix} -V'_{eL}c_\phi^2 - V_{\mu L}(1+s_\Phi^2) - \mathcal{A}_{\nu\nu}^{(\Sigma)} & 0 & -V_{eL}^{eff}s_{2\phi}/2 \\ 0 & -2V_{\mu L} - \mathcal{A}_{\nu\nu}^{(\Sigma)} & 0 \\ -V_{eL}^{eff}s_{2\phi}/2 & 0 & -V'_{eL}s_\phi^2 - V_{\mu L}(1+c_\Phi^2) - \mathcal{A}_{\nu\nu}^{(\Sigma)} \end{pmatrix},$$

$$\mathcal{A}_{\nu\nu}^{(\Sigma)} = 4\pi(a_{\nu_L\nu_L} + a_{\nu_R\nu_R})j_z - \dot{\Phi}.$$

$$\mathcal{M} = \begin{pmatrix} B_\perp & 0 & 0 \\ 0 & B_\perp & 0 \\ 0 & 0 & B_\perp \end{pmatrix} \times$$

$$\times \begin{pmatrix} 0 & \mu_{e\mu}c_\psi c_\phi + \mu_{e\tau}s_\psi c_\phi + \mu_{\mu\tau}s_\phi & \mu_{e\mu}s_\psi - \mu_{e\tau}c_\psi \\ -\mu_{e\mu}c_\psi c_\phi - \mu_{e\tau}s_\psi c_\phi - \mu_{\mu\tau}s_\phi & 0 & -\mu_{e\mu}c_\psi s_\phi - \mu_{e\tau}s_\psi s_\phi + \mu_{\mu\tau}c_\phi \\ -\mu_{e\mu}s_\psi + \mu_{e\tau}c_\psi & \mu_{e\mu}c_\psi s_\phi + \mu_{e\tau}s_\psi s_\phi - \mu_{\mu\tau}c_\phi & 0 \end{pmatrix},$$

$$\delta^{ik} = \frac{m_i^2 - m_k^2}{4E}, \qquad V_{eL}^{eff} = \sqrt{2}G_F n_e + V_{ee}^{\bar{\delta}}.$$

From Eq.(12) it follows that at such a basis choice the resonance conditions will not contain the angle ψ while the ψ-dependence will be transported to the resonance widths and the oscillation lengths.

- Our next task is to identify possible resonance conversions of the neutrino beam which travels both in the region of the coupled sunspots being the source of the solar flares. Remember, that for the resonance conversion totake place, there is a need to comply with the following requirements: (i) the resonance condition must be fulfilled; (ii) the resonance width must be nonzero; (iii) the neutrino beam must pass a distance comparable with the oscillation length. Then to verify the fulfillments of these requirements we should make the numerical estimates. With this aim in view, the experimental bounds on the values of the multipole moments should be used.

The Borexino experiments give the limits on the DMM's of the form

$$\mu_{\nu_e\nu_e} \leq 2.9 \times 10^{-11}\mu_B, \ \mu_{\nu_\mu\nu_\mu} \leq 1.5 \times 10^{-10}\mu_B, \ \mu_{\nu_\tau\nu_\tau} \leq 1.9 \times 10^{-10}\mu_B.$$

The value of the AM is connected with the charge radius through the relation

$$a_{\nu_i} = \frac{1}{6} < r^2(\nu_i) >.$$

Measuring the elastic neutrino-electron scattering at the TEXONO experiment leads to the bounds

$$-2.1 \times 10^{-32} \text{ cm}^2 \ \leq < r^2_{\nu_e} > \leq 3.3 \times 10^{-32} \text{ cm}^2.$$

Further, making numerical estimates, we shall take

$$\mu_{\nu_l\nu_{l'}} = 10^{-10}\mu_B, \qquad |a_{\nu_l\nu_{l'}}| = 3 \times 10^{-40} \text{ esu} \cdot \text{cm}^2.$$

The magnetic fields have nonpotential character

$$(\mathrm{rot}\,\mathbf{B})_z = 4\pi j_z \text{ and}$$

are characterized by the geometrical phase $\Phi(z)$ defined by the relation

$$B_x \pm iB_y = B_\perp e^{\pm i\Phi(z)}$$

and its first derivative $\dot{\Phi}(z)$. In what follows we shall adopt a simple model in which

$$\Phi(z) = \frac{\alpha\pi}{L_{mf}} z,$$

that is, the magnetic field exists over a distance L_{mf} and twists by an angle $\alpha\pi$ ($\alpha\pi/L_{mf}$ is the twist frequency). As a result we shall have

$$\dot{\Phi}(z) = \frac{\alpha\pi}{L_{mf}} = \frac{\alpha\pi}{t}.$$

The current values of oscillation parameters we need are as follows

$$\left.\begin{array}{lll} \Delta m^2_{31(23)} \simeq 2.56\times10^{-3}\ \mathrm{eV}^2, & \Delta m^2_{21} \simeq 7.87\times10^{-5}\ \mathrm{eV}^2, & \sin^2\theta_{12} \simeq 0.297, \\ \sin^2\theta_{13}\left(\Delta m^2_{31(32)} > 0\right) \simeq 0.0215, & \sin^2\theta_{13}\left(\Delta m^2_{31(32)} < 0\right) \simeq 0.0216, & \\ \sin^2\theta_{23}\left(\Delta m^2_{31(32)} > 0\right) \simeq 0.425, & \sin^2\theta_{13}\left(\Delta m^2_{31(32)} < 0\right) \simeq 0.589. & \end{array}\right\}$$

- **As the analysis shows in the Sun's conditions the Majorana neutrinos could undergo two magnetic induced resonances, namely $\nu_1'^M \leftrightarrow \overline{\nu}_2'^M$ and $\nu_2'^M \to \overline{\nu}_1'^M$**

The resonance conditions and the maximum values of the oscillation length are as follows:

$$-2\delta^{12}c_{2\omega} + V'_{eL}c_\Phi^2 + V_{\mu L}(1+s_\Phi^2) + \mathcal{A}_{\nu\nu}^{(\Sigma)} = 0$$

$$(L_{\nu_1'^M \overline{\nu}_2'^M})_{max} \simeq \frac{2\pi}{\mu_{12}^M B_\perp},$$

for $\nu_1'^M \leftrightarrow \overline{\nu}_2'^M$ and

$$2\delta^{12}c_{2\omega} + V_{eL}c_\Phi^2 + V_{\mu L}(1+s_\Phi^2) + \mathcal{A}_{\nu\nu}^{(\Sigma)} = 0,$$

$$(L_{\nu_2'^M \overline{\nu}_1'^M})_{max} \simeq \frac{2\pi}{(\mu_{e\mu}c_\psi c_\phi + \mu_{e\tau}s_\psi c_\phi + \mu_{\mu\tau}s_\phi)B_\perp}.$$

for $\nu_2'^M \to \overline{\nu}_1'^M$

- Let us show that the formulas of the three neutrino generations convert into well known ones of the two FA. Remind since we have assumed the resonance regions are well separated then we could interpret the resonances independently from each other. Next, for our purpose, we need an expression for the probability of oscillatory transitions between two neutrino states. As such, we take the expression which corresponds to the most simple case when
- $n_e, n_n, \dot{\Phi}$ and j_z are constant

$$\mathcal{P}_{\nu_\alpha \leftrightarrow \nu_\beta} = \sin^2\theta_{eff} \sin^2\left(\frac{\pi z}{L_{\nu_\alpha \nu_\beta}}\right),$$

$$\sin^2\theta_{eff} = \frac{4\mathcal{H}^2_{\alpha\beta}}{4\mathcal{H}^2_{\alpha\beta} + (\mathcal{H}^2_{\beta\beta} - \mathcal{H}^2_{\alpha\alpha})^2}, \qquad L_{\nu_\alpha\nu_\beta} = \frac{2\pi}{\sqrt{4\mathcal{H}^2_{\alpha\beta} + (\mathcal{H}^2_{\beta\beta} - \mathcal{H}^2_{\alpha\alpha})^2}}.$$

Let us assume that we determined all the transition probabilities in the bases of the ψ' states. Then, taking into account the flavor content of these states, we can find the survival probabilities of interest to us. For the electron neutrinos we obtain

$$\mathcal{P}^M_{\nu_e\nu_e} = 1 - \left\{c^2_\phi(\mathcal{P}_{\nu_1'^M \nu_2'^M} + \mathcal{P}_{\nu_1'^M \overline{\nu}_2'^M}) + s^4_\phi s^2_\psi \mathcal{P}_{\nu_1'^M \nu_2'^M} + s^2_\phi s^2_\psi \mathcal{P}_{\nu_2'^M \overline{\nu}_1'^M}\right\}.$$

When we put ϕ and ψ equal to zero, then, as would be expected, these expressions convert into the electron neutrino survival probability found in two FA

$$\mathcal{P}_{\nu_e\nu_e} = 1 - \left\{\mathcal{P}_{\nu_{eL}\nu_{\mu L}} + \mathcal{P}_{\nu_{eL}\nu_{\mu R}}\right\}.$$

Conclusions

1. The behavior of the neutrino flux in dense matter and intensive magnetic field within three neutrino generations have been considered. The investigations have be fulfilled within the LRM for the Majorana neutrinos. One was assumed that the neutrinos possess both the dipole magnetic moment and the anapole moment while the magnetic field has a twisting nature and displays nonpotential character. As the examples of magnetic field we have covered fields of the CS’s being the source of the solar flares.

2. In the Sun's conditions the possible resonance conversions of the active neutrinos have been examined. In spite of similar behavior of the neutrino beam in the Majorana and Dirac pictures there is the principal difference between these cases. It lies in the fact that in the Dirac neutrino case all magnetic-induced resonances transfer active neutrinos into sterile ones while in the Majorana neutrino case we deal with active neutrinos only. So, if the neutrino exhibits the Majorana nature, then the solar electron neutrino flux traveling through the region of the CS’s could be converted into the active right-handed neutrinos.

3. We have also demonstrated that the expressions for the survival probability of electron neutrinos found in the three neutrino generations convert into the well known expressions of the two FA provided $\phi=\psi=0$.

4. It should be stressed that investigation of the neutrino fluxes which are emitted from the stellar objects will enable us to deduce information not only about such neutrino properties as MM’s values and their nature (Dirac or Majorana) but about stellar object structure too.