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Magnetic moment of ρ-meson in point form of Poincare-invariant quantum mechanics

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INTRODUCTION

Investigation of light bound quark-antiquark systems (mesons) have always been a convenient tool for approbation of various theoretical models and approaches for studying the structure of hadrons. In the framework of the relativistic quark model based on the point form of Poincare-invariant quantum mechanics the magnetic moment of the ρ-meson is calculated taking into account the internal structure of quarks. It is shown than the model parameters obtained from the condition of matching theoretical calculation with experimental data on lepton and radiative decays leads to the results that correlate with calculation in models based on the front and instant form of the dynamics.

STATEMENT OF THE PROBLEM AND STAGES OF THE SOLUTION

Our aim:

Using values of anomalous quark magnetic moments obtain values of vector ρ-meson form-factors in the framework of point form of Poincare-invariant quantum mechanics (further PiQM)

Stages of solution:

1. Develop a technique for calculating the matrix elements of hadron $h \to h'$ transitions for obtaining the integral representations vector mesons decay constants within the framework of a point form of PiQM.
2. Using this technique to obtain integral representations of various from factors of ρ-meson.
3. Using parameters, obtained from $V(P) \to P(V)\gamma$ decay evaluate the magnetic moments μ_ρ of the quarks in the framework of PiQM.

Basic features of the model, based on PiQM

In the case of a system of two particles with the masses m_q and $m_{\bar{Q}}$ and, respectively, with 4-momentums $p_1 = \left(\omega_{m_q}(\mathrm{p}_1), \mathbf{p}_1\right)$ $p_2 = \left(\omega_{m_{\bar{Q}}}(\mathrm{p}_2), \mathbf{p}_2\right)$ basis in point form of PiQM given by

$$|\mathbf{p_1}, \lambda_1\rangle \otimes |\mathbf{p_2}, \lambda_2\rangle = |\mathbf{p_1}, \lambda_1, \mathbf{p_2}, \lambda_2\rangle \tag{1}$$

and defines a reducible representation of the Poincare group. Using the Clebsch-Gordan decomposition for the Poincare group let's construct irreducible representation that characterizes the entire system: we introduce a full momentum

$$\mathbf{P} = \mathbf{p_1} + \mathbf{p_2} \tag{2}$$

and the relative momentum $\mathbf{k}$ of two particles [1].

[1] Keister, B. D. Relativistic Hamiltonian dynamics in nuclear and particle physics/ B. D. Keister, W. N. Polyzou // Adv.Nucl.Phys.-1991.-Vol. 20.-P. 225-479.

Basic features of the model, based on PiQM

The basis of the two-particle irreducible representation is defined by the quantum numbers of the total momentum of the total angular momentum J with a projection μ, effective mass of noninteracting particles

$$M_0 = M(q\bar{Q}) = \omega_{m_q}(\mathrm{k}) + \omega_{m_{\bar{Q}}}(\mathrm{k})\ ,\ \ \omega_m(\mathrm{k}) = \sqrt{\mathrm{k}^2 + m^2}, \qquad (3)$$

where $\mathrm{k} = |\mathbf{k}|$, and two additional numbers that remove the degeneracy of this basis. As a result, the vector of a meson with momentum $\mathbf{Q}$, mass M is given by

$$\left|\mathbf{Q}, J\mu, M(q\bar{Q})\right\rangle = \int \mathrm{d}\mathbf{k} \sqrt{\frac{\omega_{m_q}(\mathrm{p}_1)\,\omega_{m_{\bar{Q}}}(\mathrm{p}_2)}{\omega_{m_q}(\mathrm{k})\,\omega_{m_{\bar{Q}}}(\mathrm{k})\,V_0}} \times$$
$$\times \sum_{\lambda_1,\lambda_2} \sum_{\nu_1,\nu_2} \Omega \left\{ \begin{smallmatrix} \ell & s & J \\ \nu_1, & \nu_2, & \mu \end{smallmatrix} \right\} (\theta_k, \phi_k)\, \Phi^J_{\ell s}(\mathrm{k}) \times \qquad (4)$$
$$\times D^{1/2}_{\lambda_1,\nu_1}(\mathbf{n}_{W_1})\, D^{1/2}_{\lambda_2,\nu_2}(\mathbf{n}_{W_2})\, |\mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2\rangle\, .$$

Basic features of the model, based on PiQM

In (4)

$$\Omega\left\{\begin{smallmatrix}\ell & s & J\\ \nu_1, & \nu_2, & \mu\end{smallmatrix}\right\}(\theta_k,\phi_k) = \mathbf{C}\left\{\begin{smallmatrix}s_1 & s_2 & s\\ \nu_1, & \nu_2, & \lambda\end{smallmatrix}\right\}\mathbf{C}\left\{\begin{smallmatrix}\ell & s & J\\ m, & \lambda, & \mu\end{smallmatrix}\right\} Y_{\ell\, m}(\theta_k,\phi_k) =$$

$$= \mathbf{C}\left\{\begin{smallmatrix}s_1 & s_2 & s\\ \nu_1, & \nu_2, & \nu_1+\nu_2\end{smallmatrix}\right\}\mathbf{C}\left\{\begin{matrix}\ell & s & J\\ \mu-(\nu_1+\nu_2), & \nu_1+\nu_2, & \mu\end{matrix}\right\} Y_{\ell\, \mu-(\nu_1+\nu_2)}(\theta_k,\phi_k)\,, \quad (5)$$

where $\mathbf{C}\left\{\begin{smallmatrix}s_1 & s_2 & s\\ \nu_1, & \nu_2, & \lambda\end{smallmatrix}\right\}$, $\mathbf{C}\left\{\begin{matrix}\ell & s & j\\ m, & \lambda, & \mu\end{matrix}\right\}$ – Clebsch-Gordan coefficients of $SU(2)$ group, $Y_{lm}(\theta_k,\phi_k)$ – the spherical functions and $D(\mathbf{n}_W)$ – Wigner rotation function.

Wave function $\Phi^J_{\ell s}(\mathrm{k})$ taking into account the number of colors of quarks N_c is normalized by the condition[2]:

$$\sum_{\ell,s}\int\limits_0^\infty \mathrm{d}\mathrm{k}\,\mathrm{k}^2\left|\Phi^J_{\ell s}(\mathrm{k})\right|^2 = N_c. \quad (6)$$

[2] Keister, B. D. Relativistic Hamiltonian dynamics in nuclear and particle physics/ B. D. Keister, W. N. Polyzou // Adv.Nucl.Phys.-1991.-Vol. 20.-P. 225-479.

Parametrization of hadronic matrix element

Matrix element of vector $h \to h'$ meson transition determined by the expressions[3]:

$$I^{\mu}_{\lambda',\lambda} = \langle \mathbf{Q}', \lambda', M' \,| J^{\mu} |\, \mathbf{Q}, \lambda, M \rangle = \frac{1}{(2\pi)^3} \frac{1}{\sqrt{2\ V_0\ M_0}} \frac{1}{\sqrt{2\ V_0'\ M_0'}} \times$$
$$\times \Big(F_1(q^2)\ (\epsilon^*(\lambda') \cdot \epsilon(\lambda))\ P^{\mu} +$$
$$+ F_2(q^2)\ (\epsilon^{\mu}(\lambda)(q \cdot \epsilon^*(\lambda')) - \epsilon^{\mu}(\lambda)(q \cdot \epsilon^*(\lambda'))) +$$
$$+ F_3(q^2)\ \frac{(\epsilon^*(\lambda') \cdot q)\ (\epsilon(\lambda) \cdot q)}{2\ M\ M'} \Big), \tag{7}$$

where $\varepsilon^{\mu}(\lambda_V)$ – polarization vector of initial and final meson and

$$q = Q' - Q,\ \ P = Q' + Q. \tag{8}$$

[3] Bakker, Bernard L. G. The Vector meson form-factor analysis in light front dynamics / Bakker, Bernard L. G., Choi, Ho-Meoyng, Ji, Chueng-Ryong // Phys. Rev. D - 2002.- Vol.65 – P. 116001

Parametrization of hadronic matrix element

For this option of parametrization charge G_C, magnetic moment G_M and quadrupole moment G_Q of vector meson determined as

$$G_C = \left(1 + \frac{2}{3}\eta\right) F_1(q^2) + \frac{2}{3}\eta\, F_2(q^2) + \frac{2}{3}\eta(1+\eta)F_3(q^2), \tag{9}$$

$$G_M = -F_2(q^2), \tag{10}$$

$$G_Q = F_1(q^2) + F_2(q^2) + (1-\eta)F_3(q^2), \tag{11}$$

where

$$\eta = Q^2 / \left(4\ M\ M'\right),\ Q^2 = -q^2. \tag{12}$$

In our assumptions transition matrix element can be expressed in a quark basis. In this case

Parametrization of hadronic matrix element

$$I^{\mu}_{\lambda',\lambda}=\frac{1}{4\pi}\frac{1}{(2\pi)^3}\frac{1}{\sqrt{V_0\ \ V_0'}}\sum_{\nu_1,\nu_1'}\ \sum_{\nu_2,\nu_2'}\int \mathrm{d}\mathbf{k}\ \ \mathrm{d}\mathbf{k}'\Phi\left(\mathrm{k},\beta^{V}_{q\bar{Q}}\right)\Phi\left(\mathrm{k}',\beta^{V}_{q\bar{Q}}\right)\times$$

$$\times \mathrm{C}\left\{\begin{array}{ccc}\frac{1}{2} & \frac{1}{2} & 1\\ \nu_1 & \nu_2 & \lambda\end{array}\right\}\mathrm{C}\left\{\begin{array}{ccc}\frac{1}{2} & \frac{1}{2} & 1\\ \nu_1' & \nu_2' & \lambda'\end{array}\right\}\left(\frac{\bar{u}_{\nu_1'}(\mathbf{k}',m_q)B^{-1}(\mathbf{u}_{\mathbf{Q}'})\Gamma^{\mu}_{q}B(\mathbf{u}_{\mathbf{Q}})u_{\nu_1}(\mathbf{k},m_q)}{\sqrt{2\ \ \omega_{m_q}(\mathrm{k}')\ \ 2\ \ \omega_{m_q}(\mathrm{k})}}\times\right.$$

$$\langle -\mathbf{k}',\nu_2'\left|U^{\dagger}(\mathbf{u}_{\mathbf{Q}'})\ \ U(\mathbf{u}_{\mathbf{Q}})\right|-\mathbf{k},\nu_2\rangle+\frac{\bar{\upsilon}_{\nu_2}(-\mathbf{k},m_{\bar{Q}})B^{-1}(\mathbf{u}_{\mathbf{Q}})\Gamma^{\mu}_{\bar{Q}}B(\mathbf{u}_{\mathbf{Q}'})\upsilon_{\nu_2'}(-\mathbf{k}',m_{\bar{Q}})}{\sqrt{2\ \ \omega_{m_{\bar{Q}}}(\mathrm{k})\ \ 2\ \ \omega_{m_{\bar{Q}}}(\mathrm{k}')}}\times$$

$$\left.\times\ \langle \mathbf{k}',\nu_1'\left|U^{\dagger}(\mathbf{u}_{\mathbf{Q}'})\ \ U(\mathbf{u}_{\mathbf{Q}})\right|\mathbf{k},\nu_1\rangle\right).$$

Expression can be simplified in Breit system, where [4]

$$\mathbf{V}_{\mathbf{Q}}+\mathbf{V}'_{\mathbf{Q}'}=0,\ \ \mathbf{V}_{\mathbf{Q}}=\{0,0,|\mathrm{V}_{Q}|\} \tag{13}$$

and

[4] Andreev,V.V., Haurysh, V.Yu. Electromagnetic form-factor of the vector mesons in the relativistic quark model / V.V. Andreev, V.Yu. Haurysh //Journal of the belarussian state university. Physics–2019.–№.1 (2019).–p.63-71.

Parametrization of hadronic matrix element

$$U^{\dagger}(\mathbf{u_{Q'}})\ U(\mathbf{u_Q}) = U(\boldsymbol{\upsilon}_{\mathbf{Q}}),\ B(\mathbf{u_{Q'}}) = B(-\mathbf{u_Q}),$$
$$\mathbf{u_Q} = \frac{\mathbf{Q}}{\omega_M(\mathrm{Q}) + M},\ \boldsymbol{\upsilon}_{\mathbf{Q}} = \frac{\mathbf{Q}}{\omega_M(\mathrm{Q})}. \tag{14}$$

Taking into account following equations

$$\mathbf{k}_{1,2} = \mathbf{k}\ \pm\ \boldsymbol{\upsilon}_{\mathbf{Q}}\left((\varpi_{1,2}+1)\omega_{m_{q,\bar{Q}}}(\mathrm{k}) - \mathrm{k}\sqrt{\varpi_{1,2}^2 - 1}\cos\theta_{\mathrm{k}}\right), \tag{15}$$

$$\mathbf{n}_{W_{2,1}} = -\frac{[\mathbf{k}, \mathbf{V_Q}]}{\omega_{m_{q,\bar{Q}}} + m_{q,\bar{Q}} - (\mathbf{k}\cdot\mathbf{V_Q})}$$

after integration over $\mathbf{k}'$ one can obtain

Parametrization of hadronic matrix element

$$I^{\mu}_{\lambda',\lambda} = \frac{1}{4\pi}\frac{1}{(2\pi)^3}\frac{1}{\sqrt{V_0 \ V_0'}} \ \sum_{\nu_1,\nu_1'} \ \sum_{\nu_2,\nu_2'} \int \ d\mathbf{k} \ \Phi\left(k,\beta^V_{q\bar{Q}}\right)\times$$

$$\times C\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \nu_1 & \nu_2 & \lambda \end{Bmatrix}\left(\frac{\bar{u}_{\nu_1'}(\mathbf{k}_2,m_q)B(\boldsymbol{\upsilon}_\mathbf{Q})\Gamma^{\mu}_{q}u_{\nu_1}(\mathbf{k},m_q)}{\sqrt{2 \ \omega_{m_q}(k_2) \ 2 \ \omega_{m_q}(k)}}\Phi\left(k_2,\beta^V_{q\bar{Q}}\right)\sqrt{\frac{\omega_{m_{\bar{Q}}}(k_2)}{\omega_{m_{\bar{Q}}}(k)}}D_{\nu_2',\nu_2}(\mathbf{n}_{W_2})+\right.$$

$$\left.+\frac{\bar{\upsilon}_{\nu_2}(\mathbf{k},m_{\bar{Q}})B(-\boldsymbol{\upsilon}_\mathbf{Q})\Gamma^{\mu}_{\bar{Q}}\upsilon_{\nu_2'}(\mathbf{k}_1,m_{\bar{Q}})}{\sqrt{2 \ \omega_{m_{\bar{Q}}}(k)2 \ \omega_{m_{\bar{Q}}}(k_1)}}\Phi\left(k_1,\beta^V_{q\bar{Q}}\right)\sqrt{\frac{\omega_{m_q}(k_1)}{\omega_{m_q}(k)}}D_{\nu_1',\nu_1}(\mathbf{n}_{W_1})\right) \ C\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \nu_1' & \nu_2' & \lambda' \end{Bmatrix}.$$

Calculation of form-factors (9) and (10) we will commit taking into account the expression

$$\Gamma^{\mu}_{q,\bar{Q}} = F_1(q^2)\gamma^{\mu} + F_2(q^2)\frac{i \ \sigma^{\mu\nu}}{2 \ m_{q,\bar{Q}}}q_{\nu}. \qquad (16)$$

Parametrization of hadronic matrix element

To fulfill the conditions of conservation of current we modify matrix element $I^{\mu}_{\lambda',\lambda}$ with

$$I^{\mu}_{\lambda',\lambda} \Rightarrow (-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2}) I_{\lambda',\lambda\ \nu}. \quad (17)$$

In Breit system polarization vectors could be chosen as

$$\epsilon(\lambda)=\frac{1}{\sqrt{2}}\{(\lambda^2-1)^2\sqrt{-1+\varpi_{1,2}},\lambda^2,-i\lambda,(\lambda^2-1)^2\sqrt{1+\varpi_{1,2}}\},$$

$$\epsilon(\lambda')=\frac{1}{\sqrt{2}}\{(\lambda'^{\ 2}-1)^2\sqrt{-1+\varpi_{1,2}},\lambda'^{\ 2},-i\lambda',-(\lambda'^{\ 2}-1)^2\sqrt{1+\varpi_{1,2}}\}$$

where $\varpi_{1,2} = (V_Q \cdot V_{Q'})$.

For different pairs of vector meson helicities the right side of (7) could be written as:

Parametrization of hadronic matrix element

Table: Form factors of vector mesons

	$\lambda = 1$	$\lambda = 0$	$\lambda = -1$
$\lambda' = 1$	$F_1(q^2) \times \ell_{F_1}$	$F_2(q^2) \times \ell_{F_2}^{(+)}$	0
$\lambda' = 0$	$F_2(q^2) \times \ell_{F_2}^{(-)}$	linear combination of $F_1(q^2), F_2(q^2)$ $F_3(q^2)$	$F_2(q^2) \times \ell_{F_2}^{(+)}$
$\lambda' = -1$	0	$F_2(q^2) \times \ell_{F_2}^{(-)}$	$F_1(q^2) \times \ell_{F_1}$

where

$$\ell_{F_1} = \frac{M+M'}{2\sqrt{M\ M'}} \left\{ -1,0,0,-\frac{M-M'}{M+M'}\frac{\sqrt{-1+\varpi_{1,2}}}{\sqrt{1+\varpi_{1,2}}} \right\},\ \ell_{F_2}^{(\pm)} = \frac{1}{2}\sqrt{\frac{M'(-1+\varpi_{1,2})}{M}}\{0,1,\pm i,0\}. \quad (18)$$

From the table one can see that form-factor $F_1(q^2)$ can be calculated directly from a matrix element $I^{\mu}_{\pm1,\pm1}$.

Calculation of $F_1(q^2)$ in PiQM

Using that in point form of PiQM

$$M = \omega_{m_q}(\mathrm{k}) + \omega_{m_{\bar{Q}}}(\mathrm{k}), \quad M' = \omega_{m_q}(\mathrm{k}_{1,2}) + \omega_{m_{\bar{Q}}}(\mathrm{k}_{1,2}) \qquad (19)$$

after some calculations of the spinor part the expression $I^{\mu}_{\pm1,\pm1}$, integration over angular variables of relative momentum $\mathbf{k}$ and limiting $q^2 \to 0$ from (9) one can obtain

$$G_C(0) = F_1(0) = \int \mathrm{dk}\ \mathrm{k}^2 \left| \Phi\left(\mathrm{k}, \beta^{V}_{q\bar{Q}}\right)\right|^2 (e_q - e_{\bar{Q}}) \,. \qquad (20)$$

For charge ρ-meson $e_u = 2/3$, $e_{\bar{d}} = -1/3$ (in units e) taking normalized condition (6) one can get $G_C(0) = 1$.

Calculation of $F_2(q^2)$ in PiQM

Calculating magnetic moment of ρ-meson could be performed directly from $I^{1,2}_{\pm1,0}$ or $I^{1,2}_{0,\pm1}$ (calculation equivalents). Similar procedure for calculating quark transition currents and limiting $q^2 \to 0$ from (10) gives

$$F_2(0) = \int \mathrm{dk}\ \mathrm{k}^2 \left|\Phi\left(\mathrm{k}, \beta^V_{q\bar{Q}}\right)\right|^2 \Big(e_q\ \eta_1(k, m_q, m_{\bar{Q}}) + \tag{21}$$

$$+\frac{e_q\kappa_q}{2m_q}\ \eta_2(k, m_q, m_{\bar{Q}}) - e_{\bar{Q}}\ \eta_1(k, m_{\bar{Q}}, m_q) - \frac{e_{\bar{Q}}\kappa_{\bar{Q}}}{2m_{\bar{Q}}}\ \eta_2(k, m_{\bar{Q}}, m_q)\Big)\ .$$

In (21) $\kappa_q, \kappa_{\bar{Q}}$ – anomalous quark magnetic moments and for $\eta_1(k, m_q, m_{\bar{Q}})$, $\eta_2(k, m_q, m_{\bar{Q}})$ we use following notations:

Calculation of $F_2(q^2)$ in PiQM

$$\eta_1(k,m_q,m_{\bar{Q}})=\frac{1}{3}\,\frac{\omega_{m_q}(\mathrm{k})\left(2\;m_q-m_{\bar{Q}}+\omega_{m_q}(\mathrm{k})\right)+\omega_{m_{\bar{Q}}}(\mathrm{k})\left(m_q+3\;\omega_{m_q}(\mathrm{k})\right)}{\omega_{m_q}^2(\mathrm{k})},$$

$$\eta_2(k,m_q,m_{\bar{Q}})=\frac{2}{3}\,\frac{\mathrm{k}^2\left(\mathrm{k}^2+\left(\omega_{m_q}(\mathrm{k})+2\;m_q\right)\left(\omega_{m_{\bar{Q}}}(\mathrm{k})+2\;m_q\right)\right)+3\;m_q^2\left(\omega_{m_q}(\mathrm{k})+m_q\right)\left(\omega_{m_{\bar{Q}}}(\mathrm{k})+m_q\right)}{\omega_{m_q}^2(\mathrm{k})\left(\omega_{m_q}(\mathrm{k})+m_q\right)}.$$

Using values of u, d-quark masses [5]

$$m_u=(219.48\pm 9.69)\ \mathrm{MeV},\ m_d=(221.97\pm 9.69)\ \mathrm{MeV},\qquad (22)$$

and anomalous magnetic moments [6]

$$\kappa_u=-0.123,\quad \kappa_d=-0.088.\qquad (23)$$

[5] Electroweak decays of unflavored mesons in Poincaré covariant quark model / V. Yu. Haurysh, V. V. Andreev // Turkish Journal of Physics. – 2019. – Vol. 43, no 2. – P. 167 – 177

[6] Poincaré-covariant quark model of electroweak light mesons decays / V. Yu. Haurysh, V. V. Andreev // The European Physics Journal (Web of Conferences). – 2019. – Vol. 204. – P. 08006.

Calculation of $F_2(q^2)$ in PiQM

numerical calculation with oscillator wave function

$$\Phi(\mathrm{k},\beta) = \frac{2}{\pi^{1/4}\ \beta^{3/2}}\ \exp\left[-\frac{\mathrm{k}^2}{2\beta^2}\right] \tag{24}$$

gives $\mu_\rho = 2.29 \pm 0.02$ with anomalous magnetic moments and $\mu_\rho = 2.91 \pm 0.02$ without κ_u, κ_d (in $e/2m_\rho$ units).

We will compare with experimental data: collaboration BaBar [7] in the energy range from 0.9 to 2.2 GeV from $e^+e^- \to \pi^+\pi^-2\pi^0$ reaction obtained $\mu_\rho = 2.1 \pm 0.5$.

[7] Determination of the magnetic dipole moment of the rho meson using 4 pion electroproduction data / Garcia Gudino, D. and Toledo Sanchez, G. // Int. J. Mod. Phys. Conf. Ser. – 2014. – Vol. 35. – P. 1460463.

Summary and discussion

The work is devoted to the calculation of the form factors of vector particles in a model based on the point form of PiQM. As a result of calculations, it was shown that the developed model for calculating the leptonic and radiative decays can be successfully used to analyze the observed of ρ-meson. The calculation results for μ_ρ are compared and in a good agreement with experimental data.

As the result of the work one can note the obtainment of a self-consistent model for describing leptonic and hadronic meson decays, based on point form of Poincare-invariant quantum mechanics.

THANK YOU FOR YOUR ATTENTION!

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