About anomalous gauge couplings investigation in electron-photon interactions on linear colliders

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The differential and total cross sections of the single gauge boson production processes in high energy electron-photon collisions are obtained within the Standard Model in leading order and next-to-leading order of the perturbative theory. The contribution of soft photon bremsstrahlung and hard photon bremsstrahlung in terms of dimensional regularization scheme was included. The corresponding anomalous gauge boson couplings were studied in the effective Lagrangian approach. The best conditions were determined for effects beyond the Standard Model registration.

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I. INTRODUCTION

The Standard Model (SM) is the most advanced theory for describing the particles interactions. However, it is obvious that the SM is not the universal theory, but only a low-energy approximation of more extensive model. Nevertheless, new effects which allow to state the existence of the "new physics" (physics beyond the SM) have not been discovered. For this reason the most theoretical investigations are realized for the construction and study of various extended gauge models. As a rule, these models have a considerable simplicity and predictive power. The most promising models for the research on linear colliders are the models including anomalous triple and quartic gauge boson interactions. From the most general view, it is possible to construct a generalized Lagrangian for this kind of interaction, limiting consideration of operators of finite dimension. This approach is called the effective Lagrangian method, which includes not only the above mentioned operators, but also the new anomalous gauge coupling (AGC) [1, 2].

This paper is devoted to the study of the triple anomalous gauge couplings for $Z^*Z\gamma$, $\gamma^*Z\gamma$ and $W^*W\gamma$ anomalous interactions, which could be studied on the base of the processes of electron-photon interactions. These processes [3–6]

$$e^-\gamma \to e^-\gamma, \tag{1}$$
$$e^-\gamma \to e^-Z, \tag{2}$$
$$e^-\gamma \to \nu_e W^-, \tag{3}$$

can be investigated with the precise accuracy on linear accelerators of new generation, such as the International Linear Collider (ILC) [7, 8].

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II. CROSS SECTIONS

A. Kinematic

We start from consideration of general process

$$e^-(p, m_e) + \gamma(k, 0) \to \quad C^-(p_1, m_c) + N^0(k_1, m_n), \tag{4}$$

where C^- and N^0 are the final particles (charged and neutral); $p(p_1)$ and $k(k_1)$ are the 4-momenta of initial(final) charged and neutral particles, correspondingly; $m_e(m_c)$ and $0(m_n)$ are their masses. The expression of the total cross section for the such type of processes can be written as follows:

$$\sigma = \frac{1}{8\pi(s - m_e^2)^2} \int |\mathcal{M}|^2 \, dQ^2, \tag{5}$$

where s and t are Mandelstam variables:

$$s = (p + k)^2, \tag{6}$$

$$t \equiv -Q^2 = (p - p_1)^2, \tag{7}$$

and $\mathcal{M}$ is the process amplitude. Integration can be performed using the following limits:

$$Q^2_\pm = \frac{(s + m_e^2)(s + m_c^2 - m_n^2) \pm (s - m_e^2)\sqrt{\lambda(s, m_c^2, m_n^2)}}{2s} - m_c^2 - m_n^2. \tag{8}$$

B. Radiative corrections

Taking into account the one-loop radiative corrections, squared amplitude can be written as follows:

$$|\mathcal{M}|^2 = |\mathcal{M}_{born}|^2 + \Re|\mathcal{M}_{born} \cdot \mathcal{M}_V^*|, \tag{9}$$

where $\mathcal{M}_{born}$ is the Born approximation amplitude and $\mathcal{M}_V$ is the amplitude including virtual radiative corrections (RC).

One-loop corrections contain unphysical ultraviolet (UV) and infrared (IR) divergences. UV-divergencies can be reduced by summation with additional counter-term diagrams contribution. The amplitude of virtual radiative corrections can be written as follows:

$$\mathcal{M}_V = \mathcal{M}_{OL} + \mathcal{M}_{CT}, \tag{10}$$

where $\mathcal{M}_{OL}$ is the one-loop contribution amplitude and $\mathcal{M}_{CT}$ is the counter-terms contribution amplitude.

Cancellation of IR-divergences is performed by taking into account the soft photon bremsstrahlung contribution. In this case the differential cross section can be written as

$$d\sigma_{soft} = \delta_{soft} \cdot d\sigma_{born}. \tag{11}$$

Here factor δ_{soft} is calculated as follows:

$$\delta_{soft} = -\frac{\alpha}{2\pi^2} \int_0^{\Delta E} \left[\frac{p^2}{(2pq)^2} + \frac{p_1^2}{(2p_1q)^2} - \frac{2pp_1}{(2pq)(2p_1q)} \right], \tag{12}$$

where α is fine structure constant and ΔE is the collider energy resolution. This energy is the minimal energy of the final bremsstrahlung photon that can be detected.

Performing integration in accordance with the dimensional regularization scheme for each of the terms in brackets, the factor δ_{soft} can be rewritten:

$$\delta_{soft} = -\frac{\alpha}{2\pi^2} \int_0^{\Delta E} \left[I(p) + I(p_1) - I(p, p_1) \right]. \tag{13}$$

The functions $I(q)$, $I(q_i, q_j)$ can be presented in the form [9, 10]:

$$I(q) = \pi \left[-\Delta^{IR} + \log \frac{4\Delta E^2}{\mu^2} + \frac{q^0}{|\vec{q}|} \log \frac{q^0 - |\vec{q}|}{q^0 + |\vec{q}|} \right], \tag{14}$$

$$I(q_i, q_j) = 2\pi \frac{\zeta^2 x_{ij}}{\zeta^2 m_i^2 - m_j^2} \left[\frac{1}{2} \log \frac{\zeta^2 m_i^2}{m_j^2} \left(-\Delta^{IR} + \log \frac{4\Delta E^2}{\mu^2} \right) + \right. \tag{15}$$

$$\left. + \left\langle \frac{1}{2} \log^2 \frac{u^0 - |\vec{u}|}{u^0 + |\vec{u}|} + \mathrm{Li}_2 \left(1 - \frac{u^0 + |\vec{u}|}{v} \right) + \mathrm{Li}_2 \left(1 - \frac{u^0 - |\vec{u}|}{v} \right) \right\rangle_{u=q_j}^{u=\zeta q_i} \right], \tag{16}$$

where μ is t'Hoft-Veltman mass parameter and

$$\zeta = \frac{x_{ij} + \sqrt{x_{ij}^2 - 4m_i^2 m_j^2}}{2m_i^2},$$

$$v = \frac{\zeta^2 m_i^2 - m_j^2}{2(\zeta q_i^0 - q_j^0)},$$

$$x_{ij} = 2q_i \cdot q_j.$$

The soft photon bremsstrahlung contribution depends on the collider energy resolution. However, in the soft photon approximation this energy should be much less than the interaction energy (including the masses of particles). To avoid this dependence one can take into account the hard photon bremsstrahlung contribution. The process (4) with additional final real photon can be rewritten as follows:

$$e^-(p, m_e) + \gamma(k, 0) \to C^-(p_1, m_c) + N^0(k_1, m_n) + \gamma(q, 0).$$

The expression of the total cross section for the such type of processes has the following form:

$$\sigma = \frac{(2\pi)^{-4}}{4(s - m_e^2)^2} \int |\mathcal{M}_{hard}|^2 \frac{dt_1 ds_1 ds_2 dt_2}{8\sqrt{-\Delta_4}}, \tag{17}$$

where s, t_1, s_1, s_2, t_2 are Mandelstam variables:

$$s = (p + k)^2, \qquad t_1 = (k - k_1)^2,$$

$$s_1 = (p_1 + q)^2; \qquad s_2 = (p_1 + k_1)^2;$$

$$t_2 = (p - q)^2.$$

The part corresponding to the localization of IR-divergence is the main contribution to the cross section of the bremsstrahlung process. Therefore, one can divide $|\mathcal{M}_{hard}|^2$ into the divergent and finite parts:

$$|\mathcal{M}_{hard}|^2 = |\mathcal{M}_{hard}^{IR}|^2 + \text{finite part}. \tag{18}$$

It is very useful, because $|\mathcal{M}_{hard}|^2$ can be factorized by squared matrix element in the Born approximation:

$$|\mathcal{M}_{hard}|^2 = \delta^{IR}_{hard}\,|\mathcal{M}_{born}|^2. \tag{19}$$

Factor δ^{IR}_{hard} depends on all new invariant and can be expressed as follows:

$$\delta^{IR}_{hard} = -\frac{\alpha}{\pi^2}\int\left(\frac{m_e^2}{(m_e^2-t_2)^2}+\frac{m_c^2}{(s_1-m_c^2)^2}-\frac{Q^2+m_e^2+m_c^2}{(m_e^2-t_2)(s_1-m_c^2)}\right)\frac{ds_1dt_2ds_2}{\sqrt{-\Delta_4}} \tag{20}$$

where Δ_4 is the Gram determinant. All kinematic boundaries can be obtained from the condition $\Delta_4 = 0$.After the integration over s_2, s_1, t_2 the invariant t_1 can be identified with t. Finally, for δ^{IR}_{hard} one can obtain [11]

$$\begin{aligned}\delta^{IR}_{hard} = &-\frac{\alpha}{2\pi}\left\langle \log\frac{4\Delta E^2 m_c^2}{(\bar{s}_1-m_c^2)^2}\left[2-\frac{1}{\beta_t}\log x_t\right]+\Re\left\{\log s_1-2\log\frac{m_c^2-t}{s_1-t}-\right.\right.\\ &-\log^2(s_1-t)+\log(\bar{s}_1-m_c^2)(2\log(s_1-t)-\log m_e^2 s_1)+\\ &\left.\left.+\log m_e^2 s_1\log\frac{m_c^2(s_1-t)}{-t}-\mathrm{Li}_2\frac{s_1}{m_c^2}+\mathrm{Li}_2\frac{s_1}{t}+2\mathrm{Li}_2\frac{s_1-t}{m_c^2-t}\right\}^{s_1=\bar{s}_1}_{s_1=m_c^2}\right\rangle\end{aligned} \tag{21}$$

with

$$\beta_t = \frac{\sqrt{\lambda(t,m_e^2,m_c^2)}}{m_e^2+m_c^2-t}, \tag{22}$$

$$x_t = \frac{1+\beta_t}{1-\beta_t}. \tag{23}$$

Summarizing of all above mentioned, one can write the expression for the differential cross section, including the lowest-order radiative corrections:

$$d\sigma = d\sigma_{born}(1+\delta^{IR}_{soft}+\delta^{IR}_{hard})+d\sigma_V, \tag{24}$$

and introduce the general notation for full relative radiative corrections

$$\delta^{full} = (d\sigma/d\sigma_{born}-1)\times 100\% \tag{25}$$

or

$$\delta^{full} = (\sigma/\sigma_{born}-1)\times 100\% \tag{26}$$

in case of the total cross sections.

C. Anomalous gauge couplings

Effective lagrangian of anomalous $WW\gamma$ interaction can be presented in following form [12]:

$$\begin{aligned}-\mathcal{L}_{WW\gamma}/e = &\, i\kappa_\gamma W^\dagger_\mu W_\nu F^{\mu\nu}+\\ &+\frac{i\lambda_\gamma}{m_W^2}W^\dagger_{\lambda\mu}W^\mu{}_\nu F^{\nu\lambda}-g_4^\gamma W^\dagger_\mu W_\nu\left(\partial^\mu A^\nu+\partial^\nu A^\mu\right)+\\ &+g_5^\gamma\varepsilon^{\mu\nu\rho\sigma}\left(W^\dagger_\mu \overleftrightarrow{\partial}_\rho W_\nu\right)A_\sigma+i\tilde{\kappa}_\gamma W^\dagger_\mu W_\nu\tilde{F}^{\mu\nu}+\frac{i\tilde{\lambda}_\gamma}{m_W^2}W^\dagger_{\lambda\mu}W^\mu{}_\nu\tilde{F}^{\nu\lambda},\end{aligned} \tag{27}$$

where F_μ is electromagnetic field tensor, W_μ is W-boson field, $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $\tilde{V}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}V^{\rho\sigma}$, $(A\overleftrightarrow{\partial}_\mu B) = A(\partial_\mu B) - B(\partial_\mu A)$.

Following eq. (27) one can put a vertex function of the form

$$\begin{aligned}\Gamma_\gamma^{\alpha\beta\mu}(q,\overline{q},P) =& \frac{\lambda_\gamma}{m_W^2}\left(q_\nu g^{\rho\alpha} - q^\rho g_\nu^\alpha\right)\left(\overline{q}_\rho g_\sigma^\beta - \overline{q}_\sigma g_\rho^\beta\right)\left(P^\sigma g^{\mu\nu} - P^\nu g^{\mu\sigma}\right) + \\ &+ \frac{\tilde{\lambda}_\gamma}{2m_W^2}\left(q_\nu g^{\rho\alpha} - q^\rho g_\nu^\alpha\right)\left(\overline{q}_\rho g_\sigma^\beta - \overline{q}_\sigma g_\rho^\beta\right)\left(P_\gamma g_\tau^\mu - P^\tau g_\gamma^\mu\right)\varepsilon^{\sigma\nu\gamma\tau} - \\ &- \Delta\kappa_\gamma\left(P^\alpha g^{\beta\mu} - P^\beta g^{\alpha\mu}\right) + \tilde{\kappa}_\gamma\varepsilon^{\alpha\beta\mu\nu}P_\nu \end{aligned} \tag{28}$$

with CP-odd (λ_γ, $\delta\kappa_\gamma$) and CP-even ($\tilde{\kappa}_\gamma$, $\tilde{\lambda}_\gamma$) AGC. In similar way vertex function of $V^*Z\gamma$ interaction can be presented as

$$\begin{aligned}\Gamma_{Z\gamma V}^{\alpha\beta\mu}(q_1,q_2,P) = \frac{s-m_V^2}{m_Z^2}&\left[h_1^V\left(q_2^\mu g^{\alpha\beta} - q_2^\alpha g^{\mu\beta}\right) + \frac{h_2^V}{m_Z^2}P^\alpha\left(P\cdot q_2 g^{\mu\beta} - q_2^\mu P^\beta\right) + \right. \\ &\left. + h_3^V\varepsilon^{\mu\alpha\beta\rho}q_{2\rho} + \frac{h_4^V}{m_Z^2}P^\alpha\varepsilon^{\mu\beta\rho\sigma}P_\rho q_{2\sigma}\right]. \end{aligned} \tag{29}$$

The couplings h_1^V and h_2^V are P-odd, h_3^V and h_3^V are P-even, all couplings are C-odd.

III. NUMERICAL ANALYSIS

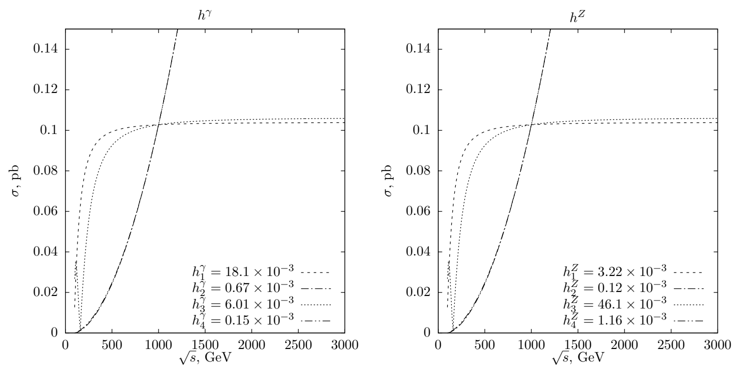
To carry out numerical analysis, some software is required. The presented results were obtained using the following tools:

- Analytical results: ***Wolfram Mathematica*** system [13];
- Squared matrix elements: ***FormCalc*** package;
- Processes kinematics: ***FeynCalc*** package [14];
- Passarino-Veltman integration: ***LoopTools*** library [15];
- Numerical integration: ***Vegas*** Monte-Carlo simulator [16].

The parametrization of unphysical UV- and IR-divergencies were performed using dimensional regularization. Final results do not depend on t'Hoft-Veltman mass regulator μ^2 and collider energy resolution ΔE. On-mass shell regularization scheme was choosed. Following experimental features were used: scattered particle angle cut $\Delta\vartheta = 20°$, ΔE is varied in the wide range of values to confirm the independence of final results from this parameter. Since the search for "new physics" implies a set of impressive experimental statistics, the analysis of the possible contribution of anomalous interactions was carried out on the basis of the total cross sections. Anomalous gauge couplings constraints were determined taking into account the following value for standard deviation σ^{SD}:

$$\sigma^{SD} = 0.001\cdot\sigma(s_0) + 1/\mathfrak{L}_{\text{int}}$$

with integrated luminosity $\mathfrak{L}_{\text{int}} = 100\ \text{pb}^{-1}$. Anomalous interactions study was performed using ND-fit. It means that only N AGC is free and other one have there SM values.



a) b)

Figure 1: The total cross section of $e\gamma \to eZ$ process obtained using $1\sigma^{SD}$ limits for h_i^γ and h_i^Z

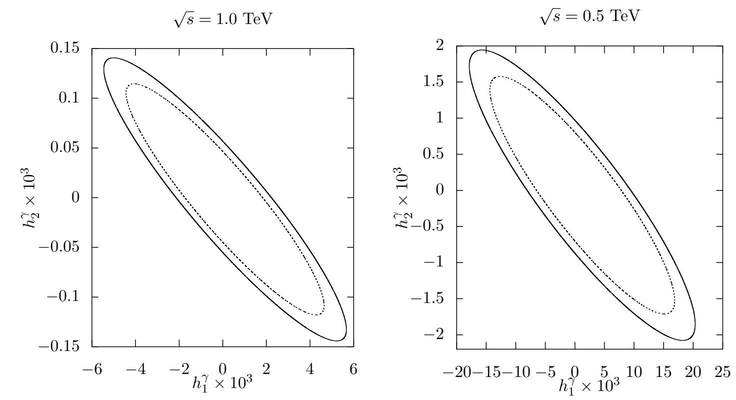


Figure 2: 99% (solid lines) and 95% (dashed lines) C.L. constraints for the $(h_1^\gamma,\, h_2^\gamma)$ gauge couplings

A. $V^*Z\gamma$ interactions

We start from the analysis of anomalous $V^*Z\gamma$ interactions behavior. In the linear approximation anomalous contribution to the total cross section of Z-boson production can be written in the following form:

$$\sigma_{eZ}^{(1)} = \sum_{i,V} h_i^V \cdot f_i^V, \tag{30}$$

where f_i^V are form-factors of corresponding AGC. In 1D-fit constraints can be derived with 68% confidence level (C.L.) from the following inequality:

$$\sigma^{SD} \geq |h_i^V \cdot f_i^V| \tag{31}$$

Contributions of terms for every $V^*Z\gamma$ coupling in 1D-fit are presented in Fig. 1. As one can see, with increasing of interaction energy the absolute value of couplings with odd indexes increases fast starting from energy value about 1.5 TeV. This behavior is explained by the presence of a gauge cancellation. The extremely important result of this analysis is that contributions of CP-odd/even couplings to the $e\gamma \to eZ$ process are indistinguishable, both qualitatively and quantitatively. This fact suggests that the 2D-fit analysis for the $(h_1^\gamma,\, h_2^\gamma)$ will be sufficient for a complete analysis of neutral AGC.

2D-fit is based on the quadratic polynom. $N\sigma^{SD}$ neutral AGC constrains have been derived from the following inequality:

$$N\sigma^{SD} \geq |\sigma_{eZ}^{(1)} + \sigma_{eZ}^{(2)}|, \tag{32}$$

where

$$\sigma_{eZ}^{(2)} = \sum_{i,j} h_i^{\gamma} h_j^{\gamma} \cdot g_{ij}. \tag{33}$$

Results of 2D-fit of 2σ and 3σ levels for different interaction energies are presented in Fig. 2. In the case of 2D-fit, the constraints are much tougher. With an increasing of the interaction energy, the obtained restrictions strongly growth. Therefore, the research of neutral AGC seems to be the most promising at the highest possible energies.

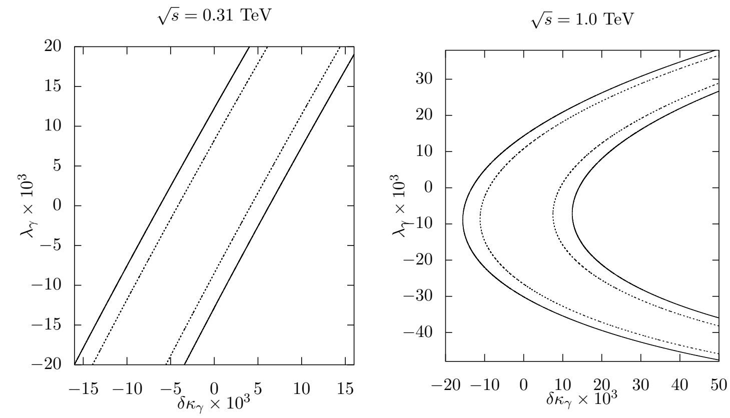


Figure 3: 99% (solid lines) and 95% (dashed lines) C.L. constraints for the ($\delta\kappa_\gamma$, λ_γ) gauge couplings

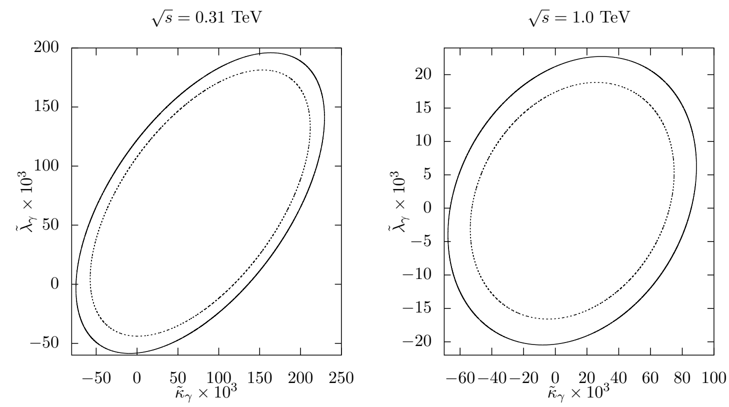


Figure 4: 99% (solid lines) and 95% (dashed lines) C.L. constraints for the $\left(\tilde{\kappa}_\gamma,\, \tilde{\lambda}_\gamma\right)$ gauge couplings

B. $W^*W\gamma$ interactions

2D-fit analysis of CP-odd/even charged AGC has been performed separately and is based on eq. (32). Restrictions on the CP-odd couplings ($\delta\kappa_\gamma$, λ_γ) depending on different interaction energies can be found in Fig. 3. As one can see, the range of possible AGC is a ring, which size decreases with increasing $\sqrt{s}$. Constraints are defined in the range near the SM values. If one compares the regions for different energy values, it will give further limit of the range of AGC.

Restrictions on the CP-even couplings $\left(\tilde{\kappa}_\gamma,\, \tilde{\lambda}_\gamma\right)$ depending on different interaction energies are presented in Fig. 4. It is easy to see that in this case, as the energy increases, it is possible to significantly refine the restrictions on the anomalous coupling constants. This behavior indicates the potential benefits of increasing the design capability of the next-generation accelerators.

IV. CONCLUSION

In the paper the differential and total cross sections of the gauge bosons production processes in electron-photon collisions including radiative corrections were calculated. Anomalous gauge boson interactions were studied. The large integral luminosity will be able significantly clarify the constraints for AGC on ILC. The numerical analysis shows that the search for manifestations of $(\delta\kappa_\gamma,\, \lambda_\gamma)$ is the best near the peak of the W-boson production. This is because the operators corresponding to these constants are 4D. Another AGC contributions set by 6D operators are significant on maximal possible interaction energy. For the $e\gamma \to eZ$ process the analysis can be performed using 2D-fit only for pair of couplings with different CP-symmetry.

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