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Spin 3/2 Particle in de Sitter Space-Time, Static Coordinates

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In de Sitter space-time, the wave equation for 16-component vector-bispinor related to spin 3/2 particle is presented in the form of three generally covariant equations, one of which can be considered as the main equations, and two additional constraints. This system ois specified in de static coordinates of the Sitter model. After separating the variable with the use of operators of energy, total angular momentum, and special reflections, we derive three system of equation with respect to 8 unknown radial function, the case of minimal value j =1/2 is simpler and it includes 6 radial functions. The main task is to find solutions of the main system, first order 8 differential equations with respect to 8 functions. This task is transformed to 2-nd order system for 4 functions, in the case of j=1/2 we have 2-nd order system for 3 functions. We have found their asymptotical behavior near the origin, r=0, and near the event horizon, r=1. Solutions in all region of the radial variable hardly can be found.

I. BASIC EQUATIONS, AND SPHERICAL SYMMETRY

In external gravitational fields, all 16 components of the wave function of a spin 3/2 particle, consisting of a vector-bispinor $\Psi_\beta(x)$, are joint in a rather complicated system of equations. We will study this system in de Sitter static cosmological model. In [1], there where constructed spherically symmetric exact solution for spin 3/2 particle in flat Minkowski model.

The complete wave equations for such a particle in de Sitter space-time may be presented in split form as a main equati0n and two additional constraints [1]:

$$[\ i\gamma^\beta(x)\ (\nabla_\beta\ +\ \Gamma_\beta(x))\ -\ m\ ]\ \Psi_\alpha(x) = 0\ , \tag{1}$$

$$\gamma^\alpha(x)\ \Psi_\alpha(x) = 0\ ,\ \ (\nabla_\alpha\ +\ \Gamma_\alpha(x))\ \Psi^\alpha(x) = 0\ . \tag{2}$$

Thew mass parameter is designated as $m = \frac{Mc}{\hbar}$; the wave function $\Psi_\alpha(x)$ is a local bispinor with respect to Lorentzian tetrad transformations, and covariant vector with respect coordinate transformations. Local Dirac matrices are determined in usual way [2], $\gamma^\beta(x) = e^\beta_{(a)}(x)\gamma^a$; and $\Gamma_\beta(x)$stands for bispinor connection [2]:

$$\Gamma_\alpha(x) = \frac{1}{2}(\sigma^{ab})_k{}^l\ e^\beta_{(a)}(\nabla_\alpha e_{(b)\beta})\ , \quad \sigma^{ab} = \frac{\gamma^a\gamma^b - \gamma^b\gamma^a}{4}\ . \tag{3}$$

Further we will apply the wave function with tetrad vector index $\Psi_l(x)$:

$$\Psi_l(x) = e^\beta_{(l)}\ \Psi_\beta(x)\ , \quad \Psi_\beta(x) = e^{(l)}_\beta\ \Psi_l(x)\ . \tag{4}$$

With (4) in mind, eq. (1) reads

$$[\ i\gamma^\alpha\ (\partial_\alpha\ +\ \Gamma_\alpha(x))\ \delta^l_k\ +\ e^\beta_{(k)}\ e^{(l)}_{\beta;\alpha})\ -\ m\ ]\ \Psi_l(x) = 0\ ; \tag{5}$$

in (5) an addition to connection Γ_α can be presented as the vector connection:

$$e^\beta_{(k)}\ e^{(l)}_{\beta;\alpha} = (L_\alpha)_k{}^l = \frac{1}{2}(j^{ab})_k{}^l\ e^\beta_{(a)}\ (\nabla_\alpha e_{(b)\beta})\ , \quad (j^{ab})_k{}^l = \delta^a_k\ g^{bl}\ -\ \delta^b_k\ g^{al}\ . \tag{6}$$

Thus, eq. (5) is re-written as

$$[\ i\gamma^\alpha(x)\ (\partial_\alpha\ +\ B_\alpha(x))\ -\ m\ ]\ \Psi(x) = 0\ , \tag{7}$$

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where $B_\alpha(x) = \Gamma_\alpha(x) \otimes I \; + \; I \otimes L_\alpha(x)$; generators for vector-bispinor representation of the Lorentz group are given by the formula $J^{ab} = \sigma^{ab} \otimes I \; + \; I \otimes j^{ab}$.

When using the function $\Psi_l(x)$, the constraints (2) take the form

$$\gamma^l \; \Psi_l(x) = 0 \; , \quad [\; e^{(l)\alpha} \; \partial_\alpha \; + \; e^{(l)\alpha}(x) \; + \; e^{(l)\alpha}(x) \; \Gamma_\alpha(x) \;] \; \Psi_l(x) = 0 \; . \tag{8}$$

Let us specify eqs. (7), (8) in static coordinates of de Sitter space-time [3]:

$$dS^2 = (1 - r^2/\rho^2) c^2 dt^2 - (1 - r^2/\rho^2)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2) \, , \tag{9}$$

where ρ stands for the curvature radius. below we will apple dimensionless quantities:

$$ct/\rho \implies t, \;\; r/\rho \implies r \; , \;\; \rho \frac{mc}{\hbar} \implies m \; . \tag{10}$$

We use the following tetrad

$$\begin{gathered} x^\alpha = (t, \; r, \; \theta, \; \phi) \; , \quad \varphi = 1 - r^2 \; , \quad \varphi' = d\varphi/dr = -2r \; , \\ e^\alpha_{(0} = (1/\sqrt{\varphi}, \; 0, \; 0, \; 0) \; , \qquad e^\alpha_{(3)} = (0, \; \sqrt{\varphi}, \; 0, \; 0) \; , \\ e^\alpha_{(1)} = (0, \; 0, \; 1/r, \; 0) \; , \quad e^\alpha_{(2)} = (0, \; 0, \; 0, \; r^{-1} \sin^{-1}\theta) \; . \end{gathered} \tag{11}$$

For local Dirac matrices $\gamma^\alpha(x)$ and connection $B_\alpha(x)$, we obtain the following expressions:

$$\begin{gathered} \gamma^\alpha(x) = (\frac{\gamma^0}{\sqrt{\varphi}}, \sqrt{\varphi}\gamma^3, \frac{\gamma^1}{r}, \frac{\gamma^2}{r\sin\theta}) \; ; \quad B_t = \frac{1}{2}\varphi' \; (\sigma^{03} \otimes I + I \otimes j^{03}) \; , \\ \Gamma_r = 0 \; , \quad \Gamma_\theta = \sqrt{\varphi} \, \sigma^{31} \; , \quad \Gamma_\phi = \sqrt{\varphi} \; \sin\theta \, \sigma^{32} \; + \; \cos\theta \, \sigma^{12} \; , \\ L_r = 0 \; , \quad L_\theta = \sqrt{\varphi} \, j^{31} \; , \quad L_\phi = \sqrt{\varphi} \; \sin\theta \, j^{32} \; + \; \cos\theta \, j^{12} \; . \end{gathered} \tag{12}$$

Correspondingly, eq. (7) takes the form

$$\left\{ i \frac{\gamma^0}{\sqrt{\varphi}} \partial_0 \; + \; i\sqrt{\varphi} \left[\gamma^3 \partial_r + \frac{\gamma^1 J^{31} + \gamma^2 J^{32}}{r} + \; \frac{\varphi'}{2\varphi} \; \gamma^0 \; (\sigma^{03} \otimes I + I \otimes j^{03}) \right] + \frac{1}{r} \Sigma_{\theta,\phi} - m \right\} \Psi(t, r, \theta, \phi) = 0 \; , \tag{13}$$

where angular operator $\Sigma_{\theta,\phi}$ is given by

$$\Sigma_{\theta,\phi} = i\gamma^1 \partial_\theta + \gamma^2 \frac{i\partial_\phi + (i\sigma^{12} \otimes I + I \otimes ij^{12}) \cos\theta}{\sin\theta} \; . \tag{14}$$

Note identity

$$\frac{i\gamma^1 J^{31} + i\gamma^2 J^{32}}{r} = \frac{1}{r} \left[i\gamma^1 (\sigma^{31} \otimes I + I \otimes j^{31}) + i\gamma^2 (\sigma^{32} \otimes I + I \otimes j^{32}) \right] = \frac{i\gamma^3}{r} + \frac{\gamma^1 \otimes T^2 - \gamma^2 \otimes T^1}{r} \; ;$$

therefore, eq. (13) can be re-written as

$$\begin{gathered} \left\{ i \frac{\gamma^0}{\sqrt{\varphi}} \partial_0 \; + \; i\sqrt{\varphi} \left[\gamma^3 (\partial_r + \frac{1}{r}) + \frac{\gamma^1 \otimes T^2 - \gamma^2 \otimes T^1}{r} + \right. \right. \\ \left. \left. + \frac{\varphi'}{2\varphi} \; \gamma^0 \; (\sigma^{03} \otimes I + I \otimes j^{03}) \right] + \frac{1}{r} \Sigma_{\theta,\phi} - m \right\} \Psi(t, r, \theta, \phi) = 0 \; , \end{gathered} \tag{15}$$

We will construct solutions with spherical symmetry, by diagonalizing the square and the third projection of the total angular momentum of the particle. In Cartesian tetrad basis we have

$$J_i^{Cart} = l_i \; + \; S_i \; , \quad S_1 = iJ^{23} \; , \qquad S_2 = iJ^{31} \; , \qquad S_3 = iJ^{12} \; . \tag{16}$$

where

$$S_i = \frac{1}{2} \; \Sigma_i \otimes I \; + \; I \otimes T_i \; , \qquad T_i = \left| \begin{array}{cc} 0 & 0 \\ 0 & \tau_i \end{array} \right| . \tag{17}$$

The wave function has two sorts of indices, bispinor A and 4-vector (l):

$$\Psi_{A(l)} = \left| \begin{array}{cccc} \Psi_{1(0)} & \Psi_{1(1)} & \Psi_{1(2)} & \Psi_{1(3)} \\ \Psi_{2(0)} & \Psi_{2(1)} & \Psi_{2(2)} & \Psi_{2(3)} \\ \Psi_{3(0)} & \Psi_{3(1)} & \Psi_{3(2)} & \Psi_{3(3)} \\ \Psi_{4(0)} & \Psi_{4(1)} & \Psi_{4(2)} & \Psi_{4(3)} \end{array} \right| . \tag{18}$$

Taking in mind 2-component representation for bispinor, we can use the notation

$$\Psi_l = \left| \begin{array}{c} \xi_{(0)}\, \delta^0_l + \xi_{(1)}\, \delta^1_l + \xi_{(2)}\, \delta^2_l + \xi_{(3)}\, \delta^3_l \\ \eta_{(0)}\, \delta^0_l + \eta_{(1)}\, \delta^1_l + \eta_{(2)}\, \delta^2_l + \eta_{(3)}\, \delta^3_l \end{array} \right| . \tag{19}$$

The wave functions in Cartesian and spherical tetrads relate to each other by local gauge transformation

$$\Psi^{sph} = S\ \Psi^{Cart} \ , \qquad S = \left| \begin{array}{cc} U_2 & 0 \\ 0 & U_2 \end{array} \right| \otimes \left| \begin{array}{cc} 1 & 0 \\ 0 & O_3 \end{array} \right| , \tag{20}$$

where $U_2(\theta,\phi)$ and $O_3(\theta,\phi)$ are given by the well-known expressions [2], [3]:

$$U_2 = \left| \begin{array}{cc} \cos\theta/2\ e^{i\phi/2} & \sin\theta/2\ e^{-i\phi/2} \\ -\sin\theta/2\ e^{i\phi/2} & \cos\theta/2\ e^{-i\phi/2} \end{array} \right| , \ (L_a{}^b) = \left| \begin{array}{cc} 1 & 0 \\ 0 & O_3 \end{array} \right| = \left| \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & \cos\theta\ \cos\phi & \cos\theta\sin\phi & -\sin\theta \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & \sin\theta\ \cos\phi & \sin\theta\ \sin\phi & \cos\theta \end{array} \right| . \tag{21}$$

Having performed the needed similarity transformation,

$$J_i^{Cart} = l_i + S_i \ , \quad J_i = S\ J_i^{Cart}\ S^{-1}$$

we derive the following explicit form for three components of the total angular momentum in spherical tetrad [3]:

$$J_1 = l_1 + S_3 \frac{\sin\phi}{\sin\theta} \ , \quad J_2 = l_2 + S_3 \frac{\cos\phi}{\sin\theta} \ , \quad J_3 = l_3 = -i \frac{\partial}{\partial\phi} \ . \tag{22}$$

Eigenvalue problems for $\vec{J}^2$ J_3:

$$\vec{J}^2\ \psi(\theta,\phi) = j(j+1)\ \psi(\theta,\phi) \, , \quad J_3\ \psi(\theta,\phi) = m\ \psi(\theta,\phi) \tag{23}$$

reduces to combining eigenfunctions in terms of Wigner D-functions [1], [3]. When that combining it is convenient to have diagonal matrix $S_3 = ij^{12}$, however it is so only in bispinor indices

$$S_3 = \frac{1}{2} \left| \begin{array}{cccc} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right| \otimes I \ + \ I \otimes \left| \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right| . \tag{24}$$

By this reason, let us perform special transformation to cyclic basis in vector space $\tilde{\Psi} = (I \otimes U)\ \Psi$:

$$\tilde{\Psi} = U\ \Psi \ , \qquad \left| \begin{array}{c} \tilde{\Psi}_{(0)} \\ \tilde{\Psi}_{(1)} \\ \tilde{\Psi}_{(2)} \\ \tilde{\Psi}_{(3)} \end{array} \right| = \left| \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & -1/\sqrt{2} & i/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & +1/\sqrt{2} & i/\sqrt{2} & 0 \end{array} \right| \left| \begin{array}{c} \Psi_{(0)} \\ \Psi_{(1)} \\ \Psi_{(2)} \\ \Psi_{(3)} \end{array} \right| ,$$

$$\Psi = U^{-1}\ \tilde{\Psi} \ , \qquad \left| \begin{array}{c} \Psi_{(0)} \\ \Psi_{(1)} \\ \Psi_{(2)} \\ \Psi_{(3)} \end{array} \right| = \left| \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -i/\sqrt{2} & 0 & -i/\sqrt{2} \\ 0 & 0 & 1 & 0 \end{array} \right| \left| \begin{array}{c} \tilde{\Psi}_{(0)} \\ \tilde{\Psi}_{(1)} \\ \tilde{\Psi}_{(2)} \\ \tilde{\Psi}_{(3)} \end{array} \right| . \tag{25}$$

This results in the needed structure

$$\tilde{S}_3 = \frac{1}{2} \left| \begin{array}{cccc} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right| \otimes I \ + \ I \otimes \left| \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right| . \tag{26}$$

In cyclic basis, eq. (7) is formally the same

$$\Big[\ i\gamma^{\alpha}(x)\ (\partial_{\alpha}\ +\ \Gamma_{\alpha}(x)\otimes I\ +\ I\otimes\tilde{L}_{\alpha}(x))\ -\ m\ \Big]\ \tilde{\Psi}(x)=0\ , \tag{27}$$

where $\tilde{L}_{\alpha}=UL_{\alpha}U^{-1}$. In cyclic basis $\tilde{\Psi}_l(x)$, additional constraints read

$$\begin{gathered}\gamma^{l}\ \Psi_{l}(x)=\gamma^{l}\ (U^{-1})_{lk}\ \tilde{\Psi}_{k}(x)=U^{*}_{kl}\ \gamma^{l}\ \tilde{\Psi}_{k}(x)=0\ ,\\ \Big[\ e^{(l)\alpha}\ \partial_{\alpha}\ +\ e^{(l)\alpha}(x)\ +\ e^{(l)\alpha}(x)\ \Gamma_{\alpha}(x)\ \Big]\ (U^{-1})_{lk}\Psi_{k}(x)=0\ .\end{gathered} \tag{28}$$

Below we will apply modified generators $\tilde{J}^{ab}$:

$$\tilde{J}^{ab}=\sigma^{ab}\otimes I\ +\ I\otimes\tilde{j}^{ab}\ ,\qquad \tilde{j}^{ab}=U\ j^{ab}\ U^{-1}\ ;$$

they are

$$i\tilde{J}^{23}=\frac{1}{\sqrt{2}}\left|\begin{array}{cccc}0&0&0&0\\0&0&1&0\\0&1&0&1\\0&0&1&0\end{array}\right|=\tilde{T}^{1}\ ,\qquad i\tilde{J}^{31}=\frac{1}{\sqrt{2}}\left|\begin{array}{cccc}0&0&0&0\\0&0&-i&0\\0&+i&0&0-i\\0&0&+i&0\end{array}\right|=\tilde{T}^{2}\ ,$$

$$i\tilde{J}^{12}=\left|\begin{array}{cccc}0&0&0&0\\0&+1&0&0\\0&0&0&0\\0&0&0&-1\end{array}\right|=\tilde{T}^{3}\ ,\qquad i\tilde{J}^{03}=\left|\begin{array}{cccc}0&0&-1&0\\0&0&0&0\\-1&0&0&0\\0&0&0&0\end{array}\right|.$$

Now turn to eq. (15). Allowing for identity $\gamma^0\sigma^{03}=\frac{1}{2}\ \gamma^3$, and separating in the wave function $\tilde{\Psi}$ a multiplier as shown $\tilde{\Psi}(x)=(\epsilon^{-i\epsilon t}/r\varphi^{1/4})\ \tilde{\Phi}(r,\theta,\phi)$, we present eq. for $\tilde{\Phi}(x)$ in the form

$$\left[\frac{\gamma^{0}}{\sqrt{\varphi}}\epsilon+i\sqrt{\varphi}\left(\gamma^{3}\partial_{r}+\frac{\gamma^{1}\otimes\tilde{T}_{2}-\gamma^{2}\otimes\tilde{T}_{1}}{r}+\frac{\varphi'}{2\varphi}\gamma^{0}\tilde{j}^{03}\right)+\frac{1}{r}\tilde{\Sigma}_{\theta,\phi}-m\right]\ \tilde{\Phi}(r,\theta,\phi)=0\ , \tag{29}$$

where

$$\tilde{\Sigma}_{\theta,\phi}=i\gamma^{1}\partial_{\theta}\ +\ \gamma^{2}\frac{i\partial_{\phi}+(i\sigma^{12}\otimes I+I\otimes i\tilde{j}^{12})\cos\theta}{\sin\theta}\ . \tag{30}$$

There exist 16 eigenstates for operators $\vec{J}^2,\ J_3$:

$$D_{-1/2}\left|\begin{array}{c}1\\0\\0\\0\end{array}\right|\delta^{0}_{l}\ ,\quad D_{-3/2}\left|\begin{array}{c}1\\0\\0\\0\end{array}\right|\delta^{1}_{l}\ ,\quad D_{-1/2}\left|\begin{array}{c}1\\0\\0\\0\end{array}\right|\delta^{2}_{l}\ ,\quad D_{+1/2}\left|\begin{array}{c}1\\0\\0\\0\end{array}\right|\delta^{3}_{l}\ ,$$

$$D_{+1/2}\left|\begin{array}{c}0\\1\\0\\0\end{array}\right|\delta^{0}_{l}\ ,\quad D_{-1/2}\left|\begin{array}{c}0\\1\\0\\0\end{array}\right|\delta^{1}_{l}\ ,\quad D_{+1/2}\left|\begin{array}{c}0\\1\\0\\0\end{array}\right|\delta^{2}_{l}\ ,\quad D_{+3/2}\left|\begin{array}{c}0\\1\\0\\0\end{array}\right|\delta^{3}_{l}\ ;$$

and 8 remaining eigenstates are similar but with other basis vectors:

$$\left|\begin{array}{c}0\\0\\1\\0\end{array}\right|,\qquad \left|\begin{array}{c}0\\0\\0\\1\end{array}\right|.$$

Therefore, the most general substitution for solutions with spherical symmetry (and quantum numbers j and m) is as follows

$$\tilde{\xi}_{l}=\left|\begin{array}{c}f_{0}\ \delta^{0}_{i}\ D_{-1/2}\ +\ f_{1}\ \delta^{1}_{i}\ D_{-3/2}\ +\ f_{2}\ \delta^{2}_{i}\ D_{-1/2}\ +\ f_{3}\ \delta^{3}_{i}\ D_{+1/2}\\ g_{0}\ \delta^{0}_{i}\ D_{+1/2}\ +\ g_{1}\ \delta^{1}_{i}\ D_{-1/2}\ +\ g_{2}\ \delta^{2}_{i}\ D_{+1/2}\ +\ g_{3}\ \delta^{3}_{i}\ D_{+3/2}\end{array}\right|; \tag{31}$$

second spin-vector $\tilde{\eta}_i$ has the similar structure but with other radial functions

$$f_i(r) \Longrightarrow h_i(r)\,, \qquad g_i(r) \Longrightarrow \nu_i(r)\,. \tag{32}$$

In explicit form they are

$$\begin{gathered}\tilde{\xi}_{(0)} = \left| \begin{array}{c} f_0(r)D_{-1/2} \\ g_0(r)D_{+1/2} \end{array} \right| , \; \tilde{\eta}_{(0)} = \left| \begin{array}{c} h_0(r)D_{-1/2} \\ \nu_0(r)D_{+1/2} \end{array} \right| , \; \tilde{\xi}_{(1)} = \left| \begin{array}{c} f_1(r)D_{-3/2} \\ g_1(r)D_{-1/2} \end{array} \right| , \quad \tilde{\eta}_{(1)} = \left| \begin{array}{c} h_1(r)D_{-3/2} \\ \nu_1(r)D_{-1/2} \end{array} \right| , \\ \tilde{\xi}_{(2)} = \left| \begin{array}{c} f_2(r)D_{-1/2} \\ g_2(r)D_{+1/2} \end{array} \right| , \; \tilde{\eta}_{(2)} = \left| \begin{array}{c} h_2(r)D_{-1/2} \\ \nu_2(r)D_{+1/2} \end{array} \right| , \; \tilde{\xi}_{(3)} = \left| \begin{array}{c} f_3(r)D_{+1/2} \\ g_3(r)D_{+3/2} \end{array} \right| , \quad \tilde{\eta}_{(3)} = \left| \begin{array}{c} h_3(r)D_{+1/2} \\ \nu_3(r)D_{+3/2} \end{array} \right| .\end{gathered} \tag{33}$$

In the case of minimal value $j = 1/2$, an initial substitution should be slightly simpler

$$\begin{gathered} j = \frac{1}{2}, \quad f_1 = 0, \quad h_1 = 0, \quad g_3 = 0, \quad \nu_3 = 0\,, \\ \tilde{\xi}_{(0)} = \left| \begin{array}{c} f_0(r)D_{-1/2} \\ g_0(r)D_{+1/2} \end{array} \right| , \quad \tilde{\eta}_{(0)} = \left| \begin{array}{c} h_0(r)D_{-1/2} \\ \nu_0(r)D_{+1/2} \end{array} \right| , \\ \tilde{\xi}_{(1)} = \left| \begin{array}{c} 0 \\ g_1(r)D_{-1/2} \end{array} \right| , \quad \tilde{\eta}_{(1)} = \left| \begin{array}{c} 0 \\ \nu_1(r)D_{-1/2} \end{array} \right| , \\ \tilde{\xi}_{(2)} = \left| \begin{array}{c} f_2(r)D_{-1/2} \\ g_2(r)D_{+1/2} \end{array} \right| , \quad \tilde{\eta}_{(2)} = \left| \begin{array}{c} h_2(r)D_{-1/2} \\ \nu_2(r)D_{+1/2} \end{array} \right| , \\ \tilde{\xi}_{(3)} = \left| \begin{array}{c} f_3(r)D_{+1/2} \\ 0 \end{array} \right| , \quad \tilde{\eta}_{(3)} = \left| \begin{array}{c} h_3(r)D_{+1/2} \\ 0 \end{array} \right| . \end{gathered} \tag{34}$$

Eq. (15) can be presented in a split form

$$\left[\frac{\epsilon}{\sqrt{\varphi}} \;+\; \sqrt{\varphi}(+i\sigma_3\partial_r \;+\; \frac{\sigma_1 \otimes \tilde{T}_2 - \sigma_2 \otimes \tilde{T}_1}{r} + i\frac{\varphi'}{2\varphi}\,\tilde{j}^{03}) \;+\; \frac{1}{r}\,\tilde{\Sigma}_{\theta,\phi} \right] \tilde{\xi} = m\,\tilde{\eta}\,, \tag{35}$$

$$\left[\frac{\epsilon}{\sqrt{\varphi}} \;+\; \sqrt{\varphi}(-i\sigma_3\partial_r \;-\; \frac{\sigma_1 \otimes \tilde{T}_2 - \sigma_2 \otimes \tilde{T}_1}{r} + i\frac{\varphi'}{2\varphi}\,\tilde{j}^{03}) \;-\; \frac{1}{r}\,\tilde{\Sigma}_{\theta,\phi} \right] \tilde{\eta} = m\,\tilde{\xi}\,, \tag{36}$$

where

$$\tilde{\Sigma}_{\theta,\phi} = i\sigma_1\partial_\theta \;+\; \sigma_2 \frac{i\partial_\phi \;+\; (1/2\sigma_3 \otimes I \;+\; I \otimes \tilde{T}_3)\cos\theta}{\sin\theta}\,. \tag{37}$$

In addition to $i\partial_t,\ \vec{J}^2,\ J_3$, we may diagonalize the operator of spacial reflection It may be found by transformation of the well-known expression in Cartesian tetrad. In particular, such an operator in vector space

$$\Pi_k^{\ l}\;\Psi_l^{Cart}(t,-\vec{r})\,, \qquad (\Pi_k^{\ l}) = \left| \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right| ,$$

in spherical tetrad and cyclic basis takes the form

$$(\tilde{\Pi}_k^{\ l}) = \left| \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right| . \tag{38}$$

P-reflection operator in bispinor space relation to spherical tetrad takes the form [3]:

$$\Pi = \left| \begin{array}{cccc} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{array} \right| . \tag{39}$$

From eigenstate equation

$$[\, (\, \Pi \otimes \tilde{\Pi}_k{}^l \,) \, \hat{P} \,] \, \tilde{\Phi}(r,\theta,\phi) = P \, \tilde{\Phi}(r,\theta,\phi) \tag{40}$$

we find two types of P-parity and related restrictions on radial functions:

$$\begin{array}{c} \nu_0 = \delta \, f_0 \, , \;\; \nu_1 = \delta \, f_3 \, , \;\; \nu_2 = \delta \, f_2 \, , \;\; \nu_3 = \delta \, f_1 \, ; \\ h_0 = \delta \, g_0 \, , \;\; h_1 = \delta \, g_3 \, , \;\; h_2 = \delta \, g_2 \, , \;\; h_3 = \delta \, g_1 \, , \end{array} \tag{41}$$

where $\delta = +1 \;\; P = (-1)^{j+1} \;\; \delta = -1 \;\; P = (-1)^{j}$.

II. SEPARATION OF THE VARIABLES IN THE MAIN EQUATION

Now we are to separate the variables in eqs. (35)–(36). First, specify the term

$$(\sigma_1 \otimes \tilde{T}_2 \; - \; \sigma_2 \otimes \tilde{T}_1)_l{}^k \, \tilde{\xi}_k \, .$$

With identities in mind

$$(\tilde{T}_2)_l{}^k \delta^0_k = 0 \, , \quad (\tilde{T}_2)_l{}^k \delta^1_k = \frac{i}{\sqrt{2}} \, \delta^2_k \, , \quad (\tilde{T}_2)_l{}^k \delta^2_k = \frac{i}{\sqrt{2}} (\delta^3_l - \delta^1_l) \, , \quad (\tilde{T}_2)_l{}^k \delta^3_k = -\frac{i}{\sqrt{2}} \delta^2_k \, ,$$

we get

$$(\sigma_1 \otimes \tilde{T}_2)_l{}^k \, \tilde{\xi}_k = \frac{i}{\sqrt{2}} \left| \begin{array}{c} g_1 \, \delta^2_l \, D_{-1/2} \; + \; g_2 \, (\delta^3_l \; - \; \delta^1_l) \, D_{+1/2} \; - \; g_3 \, \delta^2_l \, D_{+3/2} \\ f_1 \, \delta^2_l \, D_{-3/2} \; + \; f_2 \, (\delta^3_l \; - \; \delta^1_l) \, D_{-1/2} - f_3 \, \delta^2_l \, D_{+1/2} \end{array} \right| \, . \tag{42}$$

Similarly, allowing for

$$(\tilde{T}_1)_l{}^k \delta^0_k = 0 \, , \quad (\tilde{T}_1)_l{}^k \delta^1_k = \frac{1}{\sqrt{2}} \delta^2_k \, , \quad (\tilde{T}_1)_l{}^k \delta^2_k = \frac{i}{\sqrt{2}} (\delta^3_l + \delta^1_l) \, , \quad (\tilde{T}_1)_l{}^k \delta^3_k = \frac{1}{\sqrt{2}} \delta^2_k \, ,$$

we find

$$-(\sigma_2 \otimes \tilde{T}_1)_l{}^k \, \tilde{\xi}_k = \frac{i}{\sqrt{2}} \left| \begin{array}{c} g_1 \, \delta^2_l \, D_{-1/2} \; + \; g_2 \, (\delta^3_l \; + \; \delta^1_l) \, D_{+1/2} \; + \; g_3 \, \delta^2_l \, D_{+3/2} \\ -f_1 \, \delta^2_l \, D_{-3/2} \; - \; f_2 \, (\delta^3_l \; + \; \delta^1_l) \, D_{-1/2} \; - \; f_3 \, \delta^2_l \, D_{+1/2} \end{array} \right| . \tag{43}$$

Summing relations (42) and (43), we arrive at

$$(\sigma_1 \otimes \tilde{T}_2 \; - \; \sigma_2 \otimes \tilde{T}_1)_l{}^k \, \tilde{\xi}_k = i\sqrt{2} \left| \begin{array}{c} +g_1 \, \delta^2_l \, D_{-1/2} \; + \; g_2 \, \delta^3_l \, D_{+1/2} \\ -f_2 \, \delta^1_l \, D_{-1/2} \; - \; g_3 \, \delta^2_l \, D_{+1/2} \end{array} \right| . \tag{44}$$

Now, taking into account identities

$$(\tilde{j}^{03})_l{}^k \, \delta^0_k = -\delta^2_l \, , \;\; (\tilde{j}^{03})_l{}^k \, \delta^1_k = 0 \, , \;\; (\tilde{j}^{03})_l{}^k \, \delta^2_k = -\delta^0_l \, , \;\; (\tilde{j}^{03})_l{}^k \delta^3_k = 0 \, ,$$

we derive

$$i \frac{\varphi'}{2\varphi} \, (\tilde{j}^{03})_l{}^k \, \tilde{\xi}_k = i \frac{\varphi'}{2\varphi} \left| \begin{array}{c} +f_0 \, \delta^2_l \, D_{-1/2} \; + \; f_2 \, \delta^0_l \, D_{-1/2} \\ +g_0 \, \delta^2_l \, D_{+1/2} \; + \; g_2 \, \delta^0_l \, D_{+1/2} \end{array} \right| \, . \tag{45}$$

To specify the action of angular operator $\tilde{\Sigma}_{\theta,\phi}$, we are to apply the known properties for Wigner functions [3]:

$$\begin{array}{ll} \partial_\theta \, D_{+1/2} = \frac{1}{2} \left(a \, D_{-1/2} - b \, D_{+3/2} \right) , & [\sin^{-1}\theta(-m - \frac{1}{2}\cos\theta)] D_{+1/2} = \frac{1}{2} \left(-a \, D_{-1/2} - b \, D_{+3/2} \right) , \\ \partial_\theta \, D_{-1/2} = \frac{1}{2} \left(b \, D_{-3/2} - a \, D_{+1/2} \right) , & [\sin^{-1}\theta(-m + \frac{1}{2}\cos\theta)] D_{-1/2} = \frac{1}{2} \left(-b \, D_{-3/2} - a \, D_{+1/2} \right) , \\ \partial_\theta \, D_{+3/2} = \frac{1}{2} \left(b \, D_{+1/2} - c \, D_{+5/2} \right) , & [\sin^{-1}\theta(-m - \frac{3}{2}\cos\theta)] D_{+3/2} = \frac{1}{2} \left(-b \, D_{+1/2} - c \, D_{+5/2} \right) , \\ \partial_\theta \, D_{-3/2} = \frac{1}{2} \left(c \, D_{-5/2} - b \, D_{-1/2} \right) , & [\sin^{-1}\theta(-m + \frac{3}{2}\cos\theta)] D_{-3/2} = \frac{1}{2} \left(-c \, D_{-5/2} - b \, D_{-1/2} \right) , \end{array} \tag{46}$$

where

$$a = j + 1/2\,, \quad b = \sqrt{(j-1/2)(j+3/2)}\,, \quad c = \sqrt{(j-3/2)(j+5/2)}\,. \tag{47}$$

Having performed needed calculation we derive

$$(\tilde{\Sigma}_{\theta,\phi}\;\tilde{\xi})_l = i \left| \begin{array}{c} +g_0\;\delta^0_l\;a\;D_{-1/2}\;+\;g_1\;\delta^1_l\;b\;D_{-3/2}\;+\;g_2\;\delta^2_l\;a\;D_{-1/2}\;+\;g_3\;\delta^3_l\;b\;D_{+1/2} \\ -f_0\;\delta^0_l\;a\;D_{+1/2}\;-\;f_1\;\delta^1_l\;b\;D_{-1/2}\;+\;f_2\;\delta^2_l\;a\;D_{+1/2}\;-\;f_3\;\delta^3_l\;b\;D_{+3/2} \end{array} \right| . \tag{48}$$

In turn , for the term $i\sigma_3\partial_r\;\tilde{\xi}_l$ we obtain

$$i\sigma_3\partial_r\;\tilde{\xi}_l = i \left| \begin{array}{c} +f'_0\;\delta^0_l\;D_{-1/2}\;+\;f'_1\;\delta^1_l\;D_{-3/2}\;+\;f'_2\;\delta^2_l\;D_{-1/2}\;+\;f'_3\;\delta^3_l\;D_{+1/2} \\ -g'_0\;\delta^0_l\;D_{+1/2}\;-\;g'_1\;\delta^1_l\;D_{-1/2}\;-\;g'_2\;\delta^2_l\;D_{+1/2}\;-\;g'_3\;\delta^3_l\;D_{+3/2} \end{array} \right| . \tag{49}$$

Further, we find the first 8 radial equations

$$\begin{gathered}
\frac{\epsilon}{\sqrt{\varphi}} f_0 + i\sqrt{\varphi}\frac{d}{dr} f_0 - i\frac{\varphi'}{2\sqrt{\varphi}} f_2 + i\frac{a}{r} g_3 = m h_0, \frac{\epsilon}{\sqrt{\varphi}} g_0 - i\sqrt{\varphi}\frac{d}{dr} g_0 - i\frac{\varphi'}{2\sqrt{\varphi}} g_2 - i\frac{a}{r} f_0 = m\nu_0, \\
\frac{\epsilon}{\sqrt{\varphi}} f_1 + i\sqrt{\varphi}\frac{d}{dr} f_1 + i\frac{b}{r} g_1 = m h_1, \frac{\epsilon}{\sqrt{\varphi}} g_1 - i\sqrt{\varphi}\frac{d}{dr} g_1 - i\frac{\sqrt{2\varphi}}{r} f_2 - i\frac{b}{r} f_1 = m\nu_1\;, \\
\frac{\epsilon}{\sqrt{\varphi}}_2 + i\sqrt{\varphi}\frac{d}{dr} f_2 - i\frac{\varphi'}{2\sqrt{\varphi}} f_0 + i\frac{\sqrt{2\varphi}}{r} g_1 + i\frac{a}{r} g_2 = m h_2, \\
\frac{\epsilon}{\sqrt{\varphi}} g_2 - i\sqrt{\varphi}\frac{d}{dr} g_2 - i\frac{\varphi'}{2\sqrt{\varphi}} g_0 - i\frac{\sqrt{2\varphi}}{r} f_3 - i\frac{a}{r} f_2 = m\nu_2, \\
\frac{\epsilon}{\sqrt{\varphi}} f_3 + i\sqrt{\varphi}\frac{d}{dr} f_3 + i\frac{\sqrt{2\varphi}}{r} g_2 + i\frac{b}{r} g_3 = m h_3,\; \frac{\epsilon}{\sqrt{\varphi}} g_3 - i\sqrt{\varphi}\frac{d}{dr} g_3 - i\frac{b}{r} f_3 = m\nu_3.
\end{gathered} \tag{50}$$

In the new variable ω:

$$r = \sin\omega\;,\;\sqrt{\varphi} = \cos\omega\;,\;\varphi' = -2\sin\omega\;,\;\sqrt{\varphi}\frac{d}{dr} = \frac{d}{d\omega}\;. \tag{51}$$

they read as follows

$$\begin{gathered}
\frac{\epsilon}{\cos\omega} f_0 + i\frac{d}{d\omega} f_0 + i\,\mathrm{tg}\,\omega f_2 + i\frac{a}{\sin\omega} g_0 = m h_0,\;\; \frac{\epsilon}{\cos\omega} g_0 - i\frac{d}{d\omega} g_0 + i\,\mathrm{tg}\,\omega g_2 - i\frac{a}{\sin\omega} f_0 = m\nu_0, \\
\frac{\epsilon}{\cos\omega} f_1 + i\frac{d}{d\omega} f_1 + i\frac{b}{\sin\omega} g_1 = m h_1,\;\; \frac{\epsilon}{\cos\omega} g_1 - i\frac{d}{d\omega} g_1 - i\frac{\sqrt{2}}{\mathrm{tg}\,\omega} f_2 - i\frac{b}{\sin\omega} f_1 = m\nu_1, \\
\frac{\epsilon}{\cos\omega} f_2 + i\frac{d}{d\omega} f_2 + i\,\mathrm{tg}\,\omega f_0 + i\frac{\sqrt{2}}{\mathrm{tg}\,\omega} g_1 + i\frac{a}{\sin\omega} g_2 = m h_2, \\
\frac{\epsilon}{\cos\omega} g_2 - i\frac{d}{d\omega} g_2 + i\,\mathrm{tg}\,\omega g_0 - i\frac{\sqrt{2}}{\mathrm{tg}\,\omega} f_2 - i\frac{a}{\sin\omega} f_2 = m\nu_2\;, \\
\frac{\epsilon}{\cos\omega} f_3 + i\frac{d}{d\omega} f_3 + i\frac{\sqrt{2}}{\mathrm{tg}\,\omega} g_2 + i\frac{b}{\sin\omega} g_3 = m h_3,\; \frac{\epsilon}{\cos\omega} g_3 - i\frac{d}{d\omega} g_3 - i\frac{b}{\sin\omega} f_3 = m\nu_3\;.
\end{gathered} \tag{52}$$

By formal changes, from these 8 equation follows similar 8 equations for other function s

$$f_l(r) \iff h_l(r)\;,\;\; g_l(r) \iff \nu_l(r)$$

they are

$$
\begin{gathered}
\frac{\epsilon}{\cos\omega}\nu_0 + i\frac{d}{d\omega}\nu_0 + i\,\mathrm{tg}\,\omega\nu_2 + i\frac{a}{\sin\omega}h_0 = mg_0, \quad \frac{\epsilon}{\cos\omega}h_0 - i\frac{d}{d\omega}h_0 + i\mathrm{tg}\,\omega h_2 - i\frac{a}{\sin\omega}\nu_0 = mf_0, \\
\frac{\epsilon}{\cos\omega}\nu_3 + i\frac{d}{d\omega}\nu_3 + i\frac{b}{\sin\omega}h_3 = mg_3, \quad \frac{\epsilon}{\cos\omega}h_3 - i\frac{d}{d\omega}h_3 - i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}\nu_2 - i\frac{b}{\sin\omega}\nu_3 = mf_3, \\
\frac{\epsilon}{\cos\omega}\nu_2 + i\frac{d}{d\omega}\nu_2 + i\,\mathrm{tg}\,\omega\nu_0 + i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}h_3 + i\frac{a}{\sin\omega}h_2 = mg_2, \\
\frac{\epsilon}{\cos\omega}h_2 - i\frac{d}{d\omega}h_2 + i\mathrm{tg}\,\omega h_0 - i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}\nu_1 - i\frac{a}{\sin\omega}\nu_2 = mf_2, \\
\frac{\epsilon}{\cos\omega}\nu_1 + i\frac{d}{d\omega}\nu_1 + i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}h_2 + i\frac{b}{\sin\omega}h_1 = mg_1, \quad \frac{\epsilon}{\cos\omega}h_1 - i\frac{d}{d\omega}h_1 - i\frac{b}{\sin\omega}\nu_1 = mf_1\ .
\end{gathered} \tag{53}
$$

This system of 16 equations can be simplified, if one takes into account related to P-symmetry restrictions (41). In this way, for parity $(-1)^{j+1}$ we derive 8 equations for $f_i(r)$ and $g_i(r)$:

$$
\begin{gathered}
\frac{\epsilon}{\cos\omega}f_0 + i\frac{d}{d\omega}f_0 + i\mathrm{tg}\,\omega f_2 + i\frac{a}{\sin\omega}g_0 = mg_0, \quad \frac{\epsilon}{\cos\omega}g_0 - i\frac{d}{d\omega}g_0 + i\mathrm{tg}\,\omega g_2 - i\frac{a}{\sin\omega}f_0 = mf_0, \\
\frac{\epsilon}{\cos\omega}f_2 + i\frac{d}{d\omega}f_2 + i\mathrm{tg}\,\omega f_0 + i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}g_1 + i\frac{a}{\sin\omega}g_2 = mg_2, \\
\frac{\epsilon}{\cos\omega}g_2 - i\frac{d}{d\omega}g_2 + i\mathrm{tg}\,\omega g_0 - i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}f_3 - i\frac{a}{\sin\omega}f_2 = mf_2, \\
\frac{\epsilon}{\cos\omega}f_1 + i\frac{d}{d\omega}f_1 + i\frac{b}{\sin\omega}g_1 = mg_3, \quad \frac{\epsilon}{\cos\omega}g_3 - i\frac{d}{d\omega}g_3 - i\frac{b}{\sin\omega}f_3 = mf_1, \\
\frac{\epsilon}{\cos\omega}g_1 - i\frac{d}{d\omega}g_1 - i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}f_2 - i\frac{b}{\sin\omega}f_1 = mf_3, \quad \frac{\epsilon}{\cos\omega}f_3 + i\frac{d}{d\omega}f_3 + i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}g_2 + i\frac{b}{\sin\omega}g_3 = mg_1.
\end{gathered} \tag{54}
$$

For parities (-1)^j, in all equations (54) we must change m to $-m$. For the case of minimal $j = 1/2$,. the radial system (54) becomes simpler (we have to apply restrictions $f_1 = 0,\ g_3 = 0,\ b = 0,\ a = 1$):

$$
\begin{gathered}
\frac{\epsilon}{\cos\omega}f_0 + i\frac{d}{d\omega}f_0 + i\mathrm{tg}\,\omega f_2 + \frac{i}{\sin\omega}g_0 = mg_0, \quad \frac{\epsilon}{\cos\omega}g_0 - i\frac{d}{d\omega}g_0 + i\mathrm{tg}\,\omega g_2 - \frac{i}{\sin\omega}f_0 = mf_0, \\
\frac{\epsilon}{\cos\omega}f_2 + i\frac{d}{d\omega}f_2 + i\mathrm{tg}\,\omega f_0 + i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}g_1 + \frac{i}{\sin\omega}g_2 = mg_2, \\
\frac{\epsilon}{\cos\omega}g_2 - i\frac{d}{d\omega}g_2 + i\mathrm{tg}\,\omega g_0 - i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}f_3 - \frac{i}{\sin\omega}f_2 = mf_2, \\
\frac{\epsilon}{\cos\omega}g_1 - i\frac{d}{d\omega}g_1 - i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}f_2 = mf_3, \quad \frac{\epsilon}{\cos\omega}f_3 + i\frac{d}{d\omega}f_3 + i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}g_2 = mg_1.
\end{gathered} \tag{55}
$$

The main radial system (54), by combining separate equations and introducing new functions

$$
\begin{aligned}
&f_0 + g_0 = F_0, \quad f_0 - g_0 = G_0, \quad f_2 + g_2 = F_2, \quad f_2 - g_2 = G_2, \\
&f_1 + g_3 = F_1, \quad f_1 - g_3 = G_1, \quad f_3 + g_1 = F_3, \quad f_3 - g_1 = G_3,
\end{aligned} \tag{56}
$$

we can transform to different form

$$
\begin{gathered}
\frac{\epsilon}{\cos\omega}F_0 + i\frac{d}{d\omega}G_0 + i\mathrm{tg}\,\omega F_2 - i\frac{a}{\sin\omega}G_0 = mF_0, \quad \frac{\epsilon}{\cos\omega}G_0 + i\frac{d}{d\omega}F_0 + i\mathrm{tg}\,\omega G_2 + i\frac{a}{\sin\omega}F_0 = -mG_0, \\
\frac{\epsilon}{\cos\omega}F_2 + i\frac{d}{d\omega}G_2 + i\mathrm{tg}\,\omega F_0 - i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}G_3 - i\frac{a}{\sin\omega}G_2 = mF_2, \\
\frac{\epsilon}{\cos\omega}G_2 + i\frac{d}{d\omega}F_2 + i\mathrm{tg}\,\omega G_0 + i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}F_3 + i\frac{a}{\sin\omega}F_2 = -mG_2, \\
\frac{\epsilon}{\cos\omega}F_1 + i\frac{d}{d\omega}G_1 - i\frac{b}{\sin\omega}G_3 = mF_1, \quad \frac{\epsilon}{\cos\omega}G_1 + i\frac{d}{d\omega}F_1 + i\frac{b}{\sin\omega}F_3 = -mG_1, \\
\frac{\epsilon}{\cos\omega}F_3 + i\frac{d}{d\omega}G_3 - i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}G_2 - i\frac{b}{\sin\omega}G_1 = mF_3, \quad \frac{\epsilon}{\cos\omega}G_3 + i\frac{d}{d\omega}F_3 + i\frac{\sqrt{2}}{\mathrm{tg}\,\omega}F_2 + i\frac{b}{\sin\omega}F_1 = -mG_3.
\end{gathered} \tag{57}
$$

In the case of minimal $j = 1/2$, the system becomes simpler ($F_1 = 0, G_1 = 0, b = 0, a = 1$):

$$\frac{d}{d\omega}F_0 = i(\frac{\epsilon}{\cos\omega} + m)G_0 - \mathrm{tg}\,\omega G_2 - \frac{1}{\sin\omega}F_0\,, \; \frac{d}{d\omega}F_2 = i(\frac{\epsilon}{\cos\omega} + m)G_2 - \mathrm{tg}\,\omega G_0 - \frac{\sqrt{2}}{\mathrm{tg}\,\omega}F_3 - \frac{1}{\sin\omega}F_2\,,$$
$$\frac{d}{d\omega}F_3 = i(\frac{\epsilon}{\cos\omega} + m)G_3 - \frac{\sqrt{2}}{\mathrm{tg}\,\omega}F_2, \quad \frac{d}{d\omega}G_0 = i(\frac{\epsilon}{\cos\omega} - m)F_0 - \mathrm{tg}\,\omega F_2 + \frac{1}{\sin\omega}G_0\,, \qquad (58)$$
$$\frac{d}{d\omega}G_2 = i(\frac{\epsilon}{\cos\omega} - m)F_2 - \mathrm{tg}\,\omega F_0 + \frac{\sqrt{2}}{\mathrm{tg}\,\omega}G_3 + \frac{1}{\sin\omega}G_2\,, \; \frac{d}{d\omega}G_3 = i(\frac{\epsilon}{\cos\omega} - m)F_3 + \frac{\sqrt{2}}{\mathrm{tg}\,\omega}G_2\,.$$

The last system (58) for 6 functions can be written in a matrix form

$$\frac{d}{d\omega}\Psi = D\Psi\,,$$

where the the column $\Psi = \{F_0, F_2, F_3;\; G_0, G_2, G_3\}$ and (6×6)- D are used:

$$D = \left| \begin{array}{cccccc} -\frac{1}{\sin\omega} & 0 & 0 & i(\frac{\epsilon}{\cos\omega} + m) & -\tan\omega & 0 \\ 0 & -\frac{1}{\sin\omega} & -\frac{\sqrt{2}}{\tan\omega} & -\tan\omega & i(\frac{\epsilon}{\cos\omega} + m) & 0 \\ 0 & -\frac{\sqrt{2}}{\tan\omega} & 0 & 0 & 0 & i(\frac{\epsilon}{\cos\omega} + m) \\ i(\frac{\epsilon}{\cos\omega} - m) & -\tan\omega & 0 & \frac{1}{\sin\omega} & 0 & 0 \\ -\tan\omega & i(\frac{\epsilon}{\cos\omega} - m) & 0 & 0 & \frac{1}{\sin\omega} & \frac{\sqrt{2}}{\tan\omega} \\ 0 & 0 & i(\frac{\epsilon}{\cos\omega} - m) & 0 & \frac{\sqrt{2}}{\tan\omega} & 0 \end{array} \right| . \qquad (59)$$

With the use of (3×3)-blocks

$$A = \left| \begin{array}{ccc} \frac{a}{\sin\omega} & 0 & 0 \\ 0 & \frac{a}{\sin\omega} & \frac{\sqrt{2}}{\tan\omega} \\ 0 & \frac{\sqrt{2}}{\tan\omega} & 0 \end{array} \right| ,$$

$$B = \left| \begin{array}{ccc} i(\frac{\epsilon}{\cos\omega} + m) & -\tan\omega & 0 \\ -\tan\omega & +i(\frac{\epsilon}{\cos\omega} + m) & 0 \\ 0 & 0 & +i(\frac{\epsilon}{\cos\omega} + m) \end{array} \right| , \quad C = \left| \begin{array}{ccc} i(\frac{\epsilon}{\cos\omega} - m) & -\tan\omega & 0 \\ -\tan\omega & i(\frac{\epsilon}{\cos\omega} - m) & 0 \\ 0 & 0 & i(\frac{\epsilon}{\cos\omega} - m) \end{array} \right| ,$$

the above 6-dimensional equation may be presented sorter:

$$\frac{d}{d\omega}\left| \begin{array}{c} F \\ G \end{array} \right| = \left| \begin{array}{cc} -A & B \\ C & A \end{array} \right| \left| \begin{array}{c} F \\ G \end{array} \right| ,$$

which yields

$$(\frac{d}{d\omega} + A)F = BG\,, \quad (\frac{d}{d\omega} - A)G = CF\,. \qquad (60)$$

More complicated system of 8 equations (when $j = \frac{3}{2}, \frac{5}{2}, ...$) also may be presented in a matrix form

$$\frac{d}{d\omega}\Psi = D\Psi\,,$$

where the column $\Psi = \{F_0, F_1, F_2, F_3;\; G_0, G_1, G_2, G_3\}$ and 8×8 matrix D are used:

$$\left| \begin{array}{cccccccc} \frac{-a}{\sin\omega} & 0 & 0 & 0 & i(m + \frac{\epsilon}{\cos\omega}) & 0 & -\tan\omega & 0 \\ 0 & 0 & 0 & -\frac{b}{\sin\omega} & 0 & i(m + \frac{\epsilon}{\cos\omega}) & 0 & 0 \\ 0 & 0 & \frac{-a}{\sin\omega} & \frac{-\sqrt{2}}{\tan\omega} & -\tan\omega & 0 & i(m + \frac{\epsilon}{\cos\omega}) & 0 \\ 0 & \frac{-b}{\sin\omega} & \frac{-\sqrt{2}}{\tan\omega} & 0 & 0 & 0 & 0 & i(m + \frac{\epsilon}{\cos\omega}) \\ -i(m - \frac{\epsilon}{\cos\omega}) & 0 & -\tan\omega & 0 & \frac{a}{\sin\omega} & 0 & 0 & 0 \\ 0 & -i(m - \frac{\epsilon}{\cos\omega}) & 0 & 0 & 0 & 0 & 0 & \frac{b}{\sin\omega} \\ -\tan\omega & 0 & -i(m - \frac{\epsilon}{\cos\omega}) & 0 & 0 & 0 & \frac{a}{\sin\omega} & \frac{\sqrt{2}}{\tan\omega} \\ 0 & 0 & 0 & -i(m - \frac{\epsilon}{\cos\omega}) & 0 & \frac{b}{\sin\omega} & \frac{\sqrt{2}}{\tan\omega} & 0 \end{array} \right|$$

With the help of (4×4) blocks

$$A=\left|\begin{array}{cccc} \frac{a}{\sin\omega} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{b}{\sin\omega}\\ 0 & 0 & \frac{a}{\sin\omega} & \frac{\sqrt{2}}{\tan\omega}\\ 0 & \frac{b}{\sin\omega} & \frac{\sqrt{2}}{\tan\omega} & 0 \end{array}\right|,$$

$$B=\left|\begin{array}{cccc} i(m+\frac{\epsilon}{\cos\omega}) & 0 & -\tan\omega & 0\\ 0 & +i(m+\frac{\epsilon}{\cos\omega}) & 0 & 0\\ -\tan\omega & 0 & +i(m+\frac{\epsilon}{\cos\omega}) & 0\\ 0 & 0 & 0 & +i(m+\frac{\epsilon}{\cos\omega}) \end{array}\right|,$$

$$C=\left|\begin{array}{cccc} -i(m-\frac{\epsilon}{\cos\omega}) & 0 & -\tan\omega & 0\\ 0 & -i(m-\frac{\epsilon}{\cos\omega}) & 0 & 0\\ -\tan\omega & 0 & -i(m-\frac{\epsilon}{\cos\omega}) & 0\\ 0 & 0 & 0 & -i(m-\frac{\epsilon}{\cos\omega}) \end{array}\right|,$$

the matrix 8-dimensional equation is written shorter

$$\frac{d}{d\omega}\left|\begin{array}{c} F\\ G\end{array}\right| = \left|\begin{array}{cc} -A & B\\ C & A\end{array}\right| \left|\begin{array}{c} F\\ G\end{array}\right|$$

which gives

$$(\frac{d}{d\omega}+A)F=BG,\quad (\frac{d}{d\omega}-A)G=CF\,. \tag{61}$$

The structure of eqs. (60) and (61) allows for the use of exclusion method:

$$G=B^{-1}(\frac{d}{d\omega}+A)F,\quad C^{-1}(\frac{d}{d\omega}-A)B^{-1}(\frac{d}{d\omega}+A)F=F\,; \tag{62}$$

and

$$F=C^{-1}(\frac{d}{d\omega}-A)G,\quad B^{-1}(\frac{d}{d\omega}+A)C^{-1}(\frac{d}{d\omega}-A)G=G\,. \tag{63}$$

Second order equations from (62)–(63) can be reduced to different form

$$(\frac{d}{d\omega}+B\frac{dB^{-1}}{d\omega}-BAB^{-1})(\frac{d}{d\omega}+A)F=BCF\,,$$

$$(\frac{d}{d\omega}+C\frac{dC^{-1}}{d\omega}+CAC^{-1})(\frac{d}{d\omega}-A)G=CBG\,,$$

or

$$\left\{\frac{d^2}{d\omega^2}+A\frac{d}{d\omega}+\frac{dA}{d\omega}+B\frac{dB^{-1}}{d\omega}\frac{d}{d\omega}+B\frac{dB^{-1}}{d\omega}A-BAB^{-1}\frac{d}{d\omega}-BAB^{-1}A-BC\right\}F=0\,,$$

$$\left\{\frac{d^2}{d\omega^2}-A\frac{d}{d\omega}-\frac{dA}{d\omega}+C\frac{dC^{-1}}{d\omega}\frac{d}{d\omega}-C\frac{dC^{-1}}{d\omega}A+CAC^{-1}\frac{d}{d\omega}-CAC^{-1}A-CB\right\}G=0\,.$$

Thus, we arrive at two matrix-differential 2-nd order equations:

$$\left\{\frac{d^2}{d\omega^2}+\left(+A+(B\frac{dB^{-1}}{d\omega}-BAB^{-1})\right)\frac{d}{d\omega}+\left(\frac{dA}{d\omega}+(B\frac{dB^{-1}}{d\omega}-BAB^{-1})A-BC\right)\right\}F=0\,, \tag{64}$$

and

$$\left\{\frac{d^2}{d\omega^2}+\left(-A+(C\frac{dC^{-1}}{d\omega}+CAC^{-1})\right)\frac{d}{d\omega}+\left(-\frac{dA}{d\omega}-(C\frac{dC^{-1}}{d\omega}+CAC^{-1})A-CB\right)\right\}G=0\,. \tag{65}$$

III. STUDYING THE CASE $j = 1/2$

We calculate the blocks before first derivatives in (64) and (65).

$$K_1 = \left(+A + B\frac{dB^{-1}}{d\omega} - BAB^{-1} \right) = \left| \begin{matrix} -\frac{\left(e^2+m\cos\omega e+1\right)\tan\omega}{(e+m\cos\omega)^2+\sin^2\omega} & -\frac{i(m+e\cos\omega)}{(e+m\cos\omega)^2+\sin^2\omega} & -\frac{i\sqrt{2}}{m+e\sec\omega} \\ -\frac{i(m+e\cos\omega)}{(e+m\cos\omega)^2+\sin^2\omega} & -\frac{\left(e^2+m\cos\omega e+1\right)\tan\omega}{(e+m\cos\omega)^2+\sin^2\omega} & 0 \\ \frac{i\sqrt{2}(m+e\sec\omega)}{(m+e\sec\omega)^2+\tan^2\omega} & \frac{\sqrt{2}\tan\omega}{(m+e\sec\omega)^2+\tan^2\omega} & -\frac{e\tan\omega}{e+m\cos\omega} \end{matrix} \right| . \tag{66}$$

$$K_2 = \left(-A + C\frac{dC^{-1}}{d\omega} + CAC^{-1} \right) = \left| \begin{matrix} \frac{\left(-e^2+m\cos\omega e-1\right)\tan\omega}{(e-m\cos\omega)^2+\sin^2\omega} & \frac{i(m-e\cos\omega)}{(e-m\cos\omega)^2+\sin^2\omega} & -\frac{i\sqrt{2}}{m-e\sec\omega} \\ \frac{i(m-e\cos\omega)}{(e-m\cos\omega)^2+\sin^2\omega} & \frac{\left(-e^2+m\cos\omega e-1\right)\tan\omega}{(e-m\cos\omega)^2+\sin^2\omega} & 0 \\ \frac{i\sqrt{2}(m-e\sec\omega)}{(m-e\sec\omega)^2+\tan^2\omega} & -\frac{\sqrt{2}\tan\omega}{(m-e\sec\omega)^2+\tan^2\omega} & \frac{e\tan\omega}{m\cos\omega-e} \end{matrix} \right| . \tag{67}$$

Now we calculate two remaining blocks (elements of the matrices are listed by columns).

$$K_3 = \left(\frac{dA}{d\omega} + B\frac{dB^{-1}}{d\omega}A - BAB^{-1}A - BC \right) =$$

$$= \left| \begin{matrix} -m^2 + e^2\sec^2\omega - \tan^2\omega - \csc\omega(\cot\omega + \csc\omega) - \frac{\left(e^2+m\cos\omega e+1\right)\sec\omega}{(e+m\cos\omega)^2+\sin^2\omega} \\ 2ie\sec\omega\tan\omega - \frac{i(m+e\cos\omega)\csc^3\omega}{(m\cot\omega+e\csc\omega)^2+1} \\ \frac{i\sqrt{2}\csc\omega(m+e\sec\omega)}{(m+e\sec\omega)^2+\tan^2\omega} \ ; \end{matrix} \right.$$

$$= \begin{matrix} i\left(-\frac{(m+e\cos\omega)\csc^3\omega}{(m\cot\omega+e\csc\omega)^2+1} + 2e\sec\omega\tan\omega - \frac{2\cot\omega}{m+e\sec\omega} \right) \\ -m^2 - 3\csc^2\omega + e^2\sec^2\omega - \tan^2\omega - \cot\omega\csc\omega - \frac{\left(e^2+m\cos\omega e+1\right)\sec\omega}{(e+m\cos\omega)^2+\sin^2\omega} + 2 \\ \sqrt{2}\left(-\csc^2\omega - \frac{\cot\omega\csc\omega}{\frac{\sin^2\omega}{(e+m\cos\omega)^2}+1} - \frac{e}{e+m\cos\omega} \right) \ ; \end{matrix}$$

$$= \left. \begin{matrix} -\frac{i\sqrt{2}(m+e\cos\omega)\cot\omega}{(e+m\cos\omega)^2+\sin^2\omega} \\ \sqrt{2}\left(-\csc\omega(\cot\omega + \csc\omega) - \frac{\sec\omega(\sec\omega+e(m+e\sec\omega))}{(m+e\sec\omega)^2+\tan^2\omega} \right) \\ -(m+e\sec\omega)\left(\frac{2(m+e\sec\omega)\cot^2\omega}{(m+e\sec\omega)^2+\tan^2\omega} + m - e\sec\omega \right) \ . \end{matrix} \right| \tag{68}$$

$$K_4 = \left(-\frac{dA}{d\omega} - C\frac{dC^{-1}}{d\omega}A - CAC^{-1}A - CB \right) =$$

$$= \left| \begin{matrix} -m^2 + e^2\sec^2\omega - \tan^2\omega + (\cot\omega - \csc\omega)\csc\omega + \frac{\sec^2\omega\left(\left(e^2+1\right)\sec\omega - em\right)}{(m-e\sec\omega)^2+\tan^2\omega} \\ i\left(\frac{(e\cos\omega-m)\csc^3\omega}{(m\cot\omega-e\csc\omega)^2+1} + 2e\sec\omega\tan\omega \right) \\ -\frac{i\sqrt{2}\csc\omega(m-e\sec\omega)}{(m-e\sec\omega)^2+\tan^2\omega} \ ; \end{matrix} \right.$$

$$\left. \begin{matrix} i\left(\frac{(e\cos\omega-m)\csc^3\omega}{(m\cot\omega-e\csc\omega)^2+1} + 2e\sec\omega\tan\omega + \frac{2\cot\omega}{m-e\sec\omega} \right) \\ -m^2 + (\cos\omega - 3)\csc^2\omega + e^2\sec^2\omega - \tan^2\omega + \frac{\sec^2\omega\left(\left(e^2+1\right)\sec\omega - em\right)}{(m-e\sec\omega)^2+\tan^2\omega} + 2 \\ \sqrt{2}\left(\csc^2\omega\frac{\cot\omega\csc\omega}{\frac{\tan^2\omega}{(m-e\sec\omega)^2}+1} + \frac{e}{e-m\cos\omega} \right) \ ; \end{matrix} \right|$$

$$\begin{vmatrix} \frac{i\sqrt{2}(e\cos\omega-m)\cot\omega}{(e-m\cos\omega)^2+\sin^2\omega} \\ \sqrt{2}\left(\frac{\sec\omega\left(\left(e^2+1\right)\sec\omega-em\right)}{(m-e\sec\omega)^2+\tan^2\omega}+\csc\omega\tan\left(\frac{w}{2}\right)\right) \\ -(m-e\sec\omega)\left(\frac{2(m-e\sec\omega)\cot^2\omega}{(m-e\sec\omega)^2+\tan^2\omega}+m+e\sec\omega\right) \end{vmatrix}. \tag{69}$$

In order to express element of matrix blocks in rational form, let us introduce the new variable

$$\tan\frac{\omega}{2}=c, \quad \omega\in(0,\frac{\pi}{2}), x\in(0,+1)\ ; \tag{70}$$

Allowing for identities

$$\sin\omega=\frac{2x}{1+x^2},\quad \cos\omega=\frac{1-x^2}{1+x^2},\quad \tan\omega=\frac{2x}{1-x^2}\ ,\quad \sec\omega=\frac{1+x^2}{1-x^2},\quad \text{scs}\ \omega=\frac{1+x^2}{2x}\ ,$$

we find these blocks in rational form.

$$K_1=\left(+A+B\frac{dB^{-1}}{d\omega}-BAB^{-1}\right)=$$

$$=\begin{vmatrix} \frac{2x\left(x^2+1\right)\left(x^2+e\left((e-m)x^2+e+m\right)+1\right)}{(x^2-1)\left(4x^2+((e-m)x^2+e+m)^2\right)} & -\frac{i\left(x^2+1\right)\left(-ex^2+mx^2+e+m\right)}{4x^2+((e-m)x^2+e+m)^2} & \frac{i\sqrt{2}(x^2-1)}{(e-m)x^2+(e+m)} \\ -\frac{i\left(x^2+1\right)\left(-ex^2+mx^2+e+m\right)}{4x^2+((e-m)x^2+e+m)^2} & \frac{2x\left(x^2+1\right)\left(x^2+e\left((e-m)x^2+e+m\right)+1\right)}{(x^2-1)\left(4x^2+((e-m)x^2+e+m)^2\right)} & 0 \\ -\frac{i\sqrt{2}\left(x^2-1\right)\left((e-m)x^2+e+m\right)}{4x^2+((e-m)x^2+e+m)^2} & -\frac{2\sqrt{2}x\left(x^2-1\right)}{4x^2+((e-m)x^2+e+m)^2} & \frac{2ex\left(x^2+1\right)}{(x^2-1)((e-m)x^2+e+m)} \end{vmatrix}. \tag{71}$$

$$K_2=\left(-A+C\frac{dC^{-1}}{d\omega}+CAC^{-1}\right)=$$

$$=\begin{vmatrix} \frac{2x\left(x^2+1\right)\left(x^2+e\left((e+m)x^2+e-m\right)+1\right)}{(x^2-1)\left(4x^2+((e+m)x^2+e-m)^2\right)} & \frac{i\left(x^2+1\right)\left((e+m)x^2-e+m\right)}{4x^2+((e+m)x^2+e-m)^2} & -\frac{i\sqrt{2}\left(x^2-1\right)}{(e+m)x^2+e-m} \\ \frac{i\left(x^2+1\right)\left((e+m)x^2-e+m\right)}{4x^2+((e+m)x^2+e-m)^2} & \frac{2x\left(x^2+1\right)\left(x^2+e\left((e+m)x^2+e-m\right)+1\right)}{(x^2-1)\left(4x^2+((e+m)x^2+e-m)^2\right)} & 0 \\ \frac{i\sqrt{2}\left(x^2-1\right)\left((e+m)x^2+e-m\right)}{4x^2+((e+m)x^2+e-m)^2} & \frac{2\sqrt{2}x\left(x^2-1\right)}{4x^2+((e+m)x^2+e-m)^2} & \frac{2ex\left(x^2+1\right)}{(e+m)x^4-2mx^2-e+m} \end{vmatrix}. \tag{72}$$

$$K_3=\left(\frac{dA}{d\omega}+B\frac{dB^{-1}}{d\omega}A-BAB^{-1}A-BC\right)=$$

$$=\begin{vmatrix} -m^2+\frac{e^2\left(x^2+1\right)^2}{(x^2-1)^2}-\frac{x^2+1}{2x^2}+\frac{\left(x^2+1\right)^2\left(x^2+e\left((e-m)x^2+e+m\right)+1\right)}{(x^2-1)\left(4x^2+((e-m)x^2+e+m)^2\right)}-\frac{4x^2}{(x^2-1)^2} \\ i\frac{(x^2+1}{2x}\left(\frac{8ex^2}{(x^2-1)^2}+\frac{e\left(x^4-1\right)-m\left(x^2+1\right)^2}{4x^2+((e-m)x^2+e+m)^2}\right) \\ -\frac{i\left((e-m)x^2+e+m\right)\left(x^4-1\right)}{\sqrt{2}x\left(4x^2+((e-m)x^2+e+m)^2\right)}\ ; \end{vmatrix}$$

$$\begin{matrix} \frac{i}{2x}\left(\frac{8e\left(x^2+1\right)x^2}{(x^2-1)^2}-\frac{2\left(x^2-1\right)^2}{(e-m)x^2+e+m}+\frac{\left(x^2+1\right)^2\left(e\left(x^2-1\right)-m\left(x^2+1\right)\right)}{4x^2+((e-m)x^2+e+m)^2}\right) \\ -m^2-\frac{3\left(x^2+1\right)^2}{4x^2}+\frac{e^2\left(x^2+1\right)^2}{(x^2-1)^2}+\frac{x^4-1}{4x^2}+\frac{\left(x^2+1\right)^2\left(x^2+e\left((e-m)x^2+e+m\right)+1\right)}{(x^2-1)\left(4x^2+((e-m)x^2+e+m)^2\right)}-\frac{4x^2}{(x^2-1)^2}+2 \\ \frac{x^2+1}{2\sqrt{2}}\left(-\frac{4e}{(e-m)x^2+e+m}+\frac{x^2-1}{x^2}\frac{\left((e-m)x^2+e+m\right)^2}{4x^2+((e-m)x^2+e+m)^2}-\frac{1+x^2}{x^2}\right)\ ; \end{matrix}$$

$$\left.\begin{array}{c} -\frac{i\left(x^4-1\right)\left(e\left(x^2-1\right)-m\left(x^2+1\right)\right)}{\sqrt{2}x\left(4x^2+((e-m)x^2+e+m)^2\right)} \\ -\frac{\left(x^2+1\right)\left(\left(3e^2-4me+m^2+2\right)x^4+2((2e-m)(e+m)+3)x^2+(e+m)^2\right)}{\sqrt{2}x^2\left(4x^2+((e-m)x^2+e+m)^2\right)} \\ -\left(m-\frac{e\left(x^2+1\right)}{x^2-1}\right)\left(-\frac{\left((e-m)x^2+e+m\right)\left(x^2-1\right)^3}{2x^2\left(4x^2+((e-m)x^2+e+m)^2\right)}+m+\frac{e\left(x^2+1\right)}{x^2-1}\right) \end{array}\right| . \tag{73}$$

$$K_4=\left(-\frac{dA}{d\omega}-C\frac{dC^{-1}}{d\omega}A-CAC^{-1}A-CB\right)=$$

$$=\left|\begin{array}{c} \frac{e^2\left(x^2+1\right)^2}{(x^2-1)^2}-\frac{\left(x^2+1\right)^2\left(e\left(x^2(e+m)+e-m\right)+x^2+1\right)}{(x^2-1)\left((x^2(e+m)+e-m)^2+4x^2\right)}-m^2+\frac{1}{2}\left(-x^2-1\right)-\frac{4x^2}{(x^2-1)^2} \\ i\left(\frac{4ex\left(x^2+1\right)}{(x^2-1)^2}-\frac{\left(x^2+1\right)^2\left(x^2(e+m)-e+m\right)}{2x\left((x^2(e+m)+e-m)^2+4x^2\right)}\right) \\ -\frac{i\left(x^4-1\right)\left(x^2(e+m)+e-m\right)}{\sqrt{2}x\left((x^2(e+m)+e-m)^2+4x^2\right)}\ ; \end{array}\right.$$

$$\begin{array}{c} i\left(-\frac{\left(x^2-1\right)^2}{x(x^2(e+m)+e-m)}-\frac{\left(x^2+1\right)^2\left(x^2(e+m)-e+m\right)}{2x\left((x^2(e+m)+e-m)^2+4x^2\right)}+\frac{4ex\left(x^2+1\right)}{(x^2-1)^2}\right) \\ \frac{e^2\left(x^2+1\right)^2}{(x^2-1)^2}-\frac{\left(x^2+1\right)^2\left(e\left(x^2(e+m)+e-m\right)+x^2+1\right)}{(x^2-1)\left((x^2(e+m)+e-m)^2+4x^2\right)}-m^2-x^2-\frac{1}{2x^2}-\frac{4x^2}{(x^2-1)^2}+\frac{1}{2} \\ \frac{x^2+1}{2\sqrt{2}}\left(\frac{4e}{x^2(e+m)+e-m}+\frac{x^2-1}{x^2}\frac{(x^2(e+m)+e-m)^2}{4x^2+(x^2(e+m)+e-m)^2}+\frac{1+x^2}{x^2}\right)\ ; \end{array}$$

$$\left.\begin{array}{c} \frac{i\left((e+m)x^2-e+m\right)\left(x^4-1\right)}{\sqrt{2}x\left(4x^2+((e+m)x^2+e-m)^2\right)} \\ \frac{e^2\left(x^2+3\right)\left(x^2+1\right)^2+\left(6x^2+m^2\left(x^2-1\right)^2+2\right)\left(x^2+1\right)+2em\left(x^6+2x^4-x^2-2\right)}{\sqrt{2}\left(4x^2+((e+m)x^2+e-m)^2\right)} \\ -\left(m+\frac{e\left(x^2+1\right)}{x^2-1}\right)\left(\frac{\left((e+m)x^2+e-m\right)\left(x^2-1\right)^3}{2x^2\left(4x^2+((e+m)x^2+e-m)^2\right)}+m-\frac{e\left(x^2+1\right)}{x^2-1}\right) \end{array}\right| . \tag{74}$$

IV. STUDYING THE $j = 3/2, 5/2, ...$

In similar manner we find

$$K_1=\left|\begin{array}{cccc} -\frac{\tan\omega\left(e^2+em\cos\omega+1\right)}{(e+m\cos\omega)^2+\sin^2\omega} & 0 & -\frac{i(e\cos\omega+m)}{(e+m\cos\omega)^2+\sin^2\omega} & -\frac{i\sqrt{2}}{e\sec\omega+m} \\ 0 & -\frac{e\tan\omega}{e+m\cos\omega} & 0 & 0 \\ -\frac{i(e\cos\omega+m)}{(e+m\cos\omega)^2+\sin^2\omega} & 0 & -\frac{\tan\omega\left(e^2+em\cos\omega+1\right)}{(e+m\cos\omega)^2+\sin^2\omega} & 0 \\ \frac{i\sqrt{2}(e\sec\omega+m)}{(e\sec\omega+m)^2+\tan^2\omega} & 0 & \frac{\sqrt{2}\tan\omega}{(e\sec\omega+m)^2+\tan^2\omega} & -\frac{e\tan\omega}{e+m\cos\omega} \end{array}\right| . \tag{75}$$

$$K_2=\left|\begin{array}{cccc} \frac{\tan\omega)\left(-e^2+em\cos\omega-1\right)}{(e-m\cos\omega)^2+\sin^2\omega} & 0 & \frac{i(m-e\cos\omega)}{(e-m\cos\omega)^2+\sin^2\omega} & -\frac{i\sqrt{2}}{m-e\sec\omega} \\ 0 & \frac{e\tan\omega}{m\cos\omega-e} & 0 & 0 \\ \frac{i(m-e\cos\omega)}{(e-m\cos\omega)^2+\sin^2\omega} & 0 & \frac{\tan\omega\left(-e^2+em\cos\omega-1\right)}{(e-m\cos\omega)^2+\sin^2\omega} & 0 \\ \frac{i\sqrt{2}(m-e\sec\omega)}{(m-e\sec\omega)^2+\tan^2\omega} & 0 & -\frac{\sqrt{2}\tan\omega}{(m-e\sec\omega)^2+\tan^2\omega} & \frac{e\tan\omega}{m\cos\omega-e} \end{array}\right| . \tag{76}$$

Then we find the blocks K_3, K_4.

$$K_3=\left|\begin{array}{c} -a^2\csc^2\omega-\frac{a\sec\omega\left(e^2+em\cos\omega+1\right)}{(e+m\cos\omega)^2+\sin^2\omega}-a\cot\omega\csc\omega+e^2\sec^2\omega-m^2-\tan^2\omega \\ 0 \\ 2ie\tan\omega\sec\omega-\frac{ia\csc^3\omega(e\cos\omega+m)}{(e\csc\omega+m\cot\omega)^2+1} \\ \frac{i\sqrt{2}a\csc\omega(e\sec\omega+m)}{(e\sec\omega+m)^2+\tan^2\omega}\ ; \end{array}\right.$$

$$-\frac{i\sqrt{2}b\cot\omega}{e+m\cos\omega}$$
$$-b^2\csc^2(\omega)+e^2\sec^2\omega-m^2$$
$$-\sqrt{2}b\cot\omega\csc\omega$$
$$-\frac{be\sec\omega}{e+m\cos\omega}-b\cot\omega\csc\omega\ ;$$

$$i\left(-\frac{a\csc^3\omega(e\cos\omega+m)}{(e\csc\omega+m\cot\omega)^2+1}-\frac{2\cot\omega}{e\sec\omega+m}+2e\tan\omega\sec\omega\right)$$
$$-\sqrt{2}b\cot\omega\csc\omega$$
$$-\left(a^2+2\right)\csc^2\omega-\frac{a\sec\omega\left(e^2+em\cos\omega+1\right)}{(e+m\cos\omega)^2+\sin^2\omega}-a\cot\omega\csc\omega+e^2\sec^2\omega-m^2-\tan^2\omega+2$$
$$\sqrt{2}\left(-\frac{a\cot\omega\csc\omega(e\sec\omega+m)^2}{(e\sec\omega+m)^2+\tan^2\omega}-\frac{e}{e+m\cos\omega}-\csc^2\omega\right)\ ;$$

$$\left.\begin{array}{c} -\frac{i\sqrt{2}\cot\omega(e\cos\omega+m)}{(e+m\cos\omega)^2+\sin^2\omega} \\ -\frac{be\sec\omega}{e+m\cos\omega}-b\cot\omega\csc\omega \\ \sqrt{2}\left(-\csc\omega(a\cot\omega+\csc\omega)-\frac{\sec\omega(e(e\sec\omega+m)+\sec\omega)}{(e\sec\omega+m)^2+\tan^2\omega}\right) \\ -b^2\csc^2\omega+e^2\sec^2\omega-\frac{2\csc^2\omega(e+m\cos\omega)^2}{(e\sec\omega+m)^2+\tan^2\omega}-m^2 \end{array}\right| . \tag{77}$$

$$K_4=\left|\begin{array}{c} -\frac{a\sec^2\omega(e(m-e\sec\omega)-\sec\omega)}{(m-e\sec\omega)^2+\tan^2\omega}+a\csc^2\omega(\cos\omega-a)-(m-e\sec\omega)(e\sec\omega+m)-\tan^2\omega \\ 0 \\ \frac{a\sec^2\omega(-i\csc\omega(m-e\sec\omega)-ie\tan\omega)}{(m-e\sec\omega)^2+\tan^2\omega}+\tan\omega(i(e\sec\omega+m)-i(m-e\sec\omega)) \\ -\frac{i\sqrt{2}a\csc\omega(m-e\sec\omega)}{(m-e\sec\omega)^2+\tan^2\omega}\ ; \end{array}\right.$$

$$\frac{i\sqrt{2}b\csc\omega}{m-e\sec\omega}$$
$$-b^2\csc^2\omega-(m-e\sec\omega)(e\sec\omega+m)$$
$$-\sqrt{2}b\cot\omega\csc\omega$$
$$b\cot\omega\csc\omega-\frac{be\sec^2\omega}{m-e\sec\omega}\ ;$$

$$\frac{a\sec^2\omega(-i\csc\omega(m-e\sec\omega)-ie\tan\omega)}{(m-e\sec\omega)^2+\tan^2\omega}+\frac{2i\cot\omega}{m-e\sec\omega}+\tan\omega(i(e\sec\omega+m)-i(m-e\sec\omega))$$
$$-\sqrt{2}b\cot\omega\csc\omega$$
$$-\left(a^2+2\right)\csc^2\omega-\frac{a\sec^2\omega(e(m-e\sec\omega)-\sec\omega)}{(m-e\sec\omega)^2+\tan^2\omega}+a\cot\omega\csc\omega-(m-e\sec\omega)(e\sec\omega+m)-\tan^2\omega+2$$
$$\sqrt{2}\left(-\frac{a\cot\omega\csc\omega(m-e\sec\omega)^2}{(m-e\sec\omega)^2+\tan^2\omega}-\frac{e\sec\omega}{m-e\sec\omega}+\csc^2\omega\right)\ ;$$

$$\left.\begin{array}{c} \frac{\sqrt{2}\sec\omega(-i\csc\omega(m-e\sec\omega)-ie\tan\omega)}{(m-e\sec\omega)^2+\tan^2\omega} \\ b\cot\omega\csc\omega-\frac{be\sec^2\omega}{m-e\sec\omega} \\ \sqrt{2}\left(-a\cot\omega\csc\omega-\frac{\sec\omega(e(m-e\sec\omega)-\sec\omega)}{(m-e\sec\omega)^2+\tan^2\omega}+\csc^2\omega\right) \\ -b^2\csc^2\omega-(m-e\sec\omega)(e\sec\omega+m)-\frac{2\cot^2\omega(m-e\sec\omega)^2}{(m-e\sec\omega)^2+\tan^2 w} \end{array}\right| . \tag{78}$$

Then we transform them to rational form.

$$K_1=\left|\begin{array}{cc} \frac{2x\left(x^2+1\right)\left(e\left(x^2(e-m)+e+m\right)+x^2+1\right)}{(x^2-1)\left((x^2(e-m)+e+m)^2+4x^2\right)} & 0 \\ 0 & \frac{2ex\left(x^2+1\right)}{(x^2-1)(x^2(e-m)+e+m)} \\ -\frac{i\left(x^2+1\right)\left(-ex^2+e+mx^2+m\right)}{(x^2(e-m)+e+m)^2+4x^2} & 0 \\ -\frac{i\sqrt{2}\left(x^2-1\right)\left(x^2(e-m)+e+m\right)}{(x^2(e-m)+e+m)^2+4x^2} & 0 \end{array}\right.$$

$$\left.\begin{array}{cc}
-\frac{i\left(x^2+1\right)\left(-ex^2+e+mx^2+m\right)}{(x^2(e-m)+e+m)^2+4x^2} & -\frac{i\sqrt{2}}{m-\frac{e(x^2+1)}{x^2-1}} \\
0 & 0 \\
\frac{2x\left(x^2+1\right)\left(e\left(x^2(e-m)+e+m\right)+x^2+1\right)}{(x^2-1)\left((x^2(e-m)+e+m)^2+4x^2\right)} & 0 \\
-\frac{2\sqrt{2}x\left(x^2-1\right)}{(x^2(e-m)+e+m)^2+4x^2} & \frac{2ex\left(x^2+1\right)}{(x^2-1)(x^2(e-m)+e+m)}
\end{array}\right| . \tag{79}$$

$$K_2 = \left|\begin{array}{cc}
\frac{2x\left(x^2+1\right)\left(e\left(x^2(e+m)+e-m\right)+x^2+1\right)}{(x^2-1)\left((x^2(e+m)+e-m)^2+4x^2\right)} & 0 \\
0 & \frac{2ex\left(x^2+1\right)}{e(x^4-1)+m(x^2-1)^2} \\
\frac{i\left(x^2+1\right)\left(x^2(e+m)-e+m\right)}{(x^2(e+m)+e-m)^2+4x^2} & 0 \\
\frac{i\sqrt{2}\left(x^2-1\right)\left(x^2(e+m)+e-m\right)}{(x^2(e+m)+e-m)^2+4x^2} & 0
\end{array}\right.$$

$$= \left.\begin{array}{cc}
\frac{i\left(x^2+1\right)\left(x^2(e+m)-e+m\right)}{(x^2(e+m)+e-m)^2+4x^2} & -\frac{i\sqrt{2}\left(x^2-1\right)}{x^2(e+m)+e-m} \\
0 & 0 \\
\frac{2x\left(x^2+1\right)\left(e\left(x^2(e+m)+e-m\right)+x^2+1\right)}{(x^2-1)\left((x^2(e+m)+e-m)^2+4x^2\right)} & 0 \\
\frac{2\sqrt{2}x\left(x^2-1\right)}{(x^2(e+m)+e-m)^2+4x^2} & \frac{2ex\left(x^2+1\right)}{e(x^4-1)+m(x^2-1)^2}
\end{array}\right| . \tag{80}$$

$$K_3 = \left|\begin{array}{c}
-\frac{a^2(x^2+1)^2}{4x^2} + \frac{a(x^2+1)^2\left(e\left(x^2(e-m)+e+m\right)+x^2+1\right)}{(x^2-1)\left((x^2(e-m)+e+m)^2+4x^2\right)} + \frac{a\left(x^4-1\right)}{4x^2} + \frac{e^2\left(x^2+1\right)^2}{(x^2-1)^2} - m^2 - \frac{4x^2}{(x^2-1)^2} \\
0 \\
i\frac{x^2+1}{2}\left(\frac{a\left(x^2+1\right)\left(e\left(x^2-1\right)-m\left(x^2+1\right)\right)}{(x^2(e-m)+e+m)^2+4x^2} + \frac{8ex^2}{(x^2-1)^2}\right) \\
-\frac{ia\left(x^4-1\right)\left(x^2(e-m)+e+m\right)}{\sqrt{2}x\left((x^2(e-m)+e+m)^2+4x^2\right)}
\end{array}\right. ;$$

$$\begin{array}{c}
\frac{ib\left(x^4-1\right)}{\sqrt{2}x(x^2(e-m)+e+m)} \\
-\frac{b^2\left(x^2+1\right)^2}{4x^2} + \frac{e^2\left(x^2+1\right)^2}{(x^2-1)^2} - m^2 \\
\frac{b\left(x^4-1\right)}{2\sqrt{2}x^2} \\
\frac{b\left(x^2+1\right)\left(e\left(x^2+1\right)^3-m\left(x^2-1\right)^3\right)}{4x^2(x^2-1)(x^2(e-m)+e+m)}
\end{array} ;$$

$$\begin{array}{c}
\frac{i}{2x}\left(\frac{a\left(x^2+1\right)^2\left(e\left(x^2-1\right)-m\left(x^2+1\right)\right)}{(x^2(e-m)+e+m)^2+4x^2} - \frac{2\left(x^2-1\right)^2}{x^2(e-m)+e+m} + \frac{8e\left(x^2+1\right)x^2}{(x^2-1)^2}\right) \\
\frac{b\left(x^4-1\right)}{2\sqrt{2}x^2} \\
-\frac{\left(a^2+2\right)\left(x^2+1\right)^2}{4x^2} + \frac{a\left(x^2+1\right)^2\left(e\left(x^2(e-m)+e+m\right)+x^2+1\right)}{(x^2-1)\left((x^2(e-m)+e+m)^2+4x^2\right)} + \frac{a\left(x^4-1\right)}{4x^2} + \frac{e^2\left(x^2+1\right)^2}{(x^2-1)^2} - m^2 - \frac{4x^2}{(x^2-1)^2} + 2 \\
\frac{x^2+1}{2\sqrt{2}}\left(\frac{a\left(x^2-1\right)\left(x^2(e-m)+e+m\right)^2}{2x^4(e^2-m^2+2)+x^6(e-m)^2+x^2(e+m)^2} - \frac{4e}{x^2(e-m)+e+m} - \frac{1}{x^2} - 1\right)
\end{array} ; \tag{81}$$

$$\left.\begin{array}{c}
-\frac{i\left(x^4-1\right)\left(e\left(x^2-1\right)-m\left(x^2+1\right)\right)}{\sqrt{2}x\left((x^2(e-m)+e+m)^2+4x^2\right)} \\
\frac{b\left(x^2+1\right)\left(e\left(x^2+1\right)^3-m\left(x^2-1\right)^3\right)}{4x^2(x^2-1)(x^2(e-m)+e+m)} \\
\frac{x^2+1}{\sqrt{2}}\left(\frac{-2e\left(x^2(e-m)+e+m\right)-2\left(x^2+1\right)}{(x^2(e-m)+e+m)^2+4x^2} - \frac{-ax^2+a+x^2+1}{2x^2}\right) \\
-\frac{b^2\left(x^2+1\right)^2}{4x^2} + \frac{e^2\left(x^2+1\right)^2}{(x^2-1)^2} - \frac{\left(x^2-1\right)^2\left(x^2(e-m)+e+m\right)^2}{2x^2\left((x^2(e-m)+e+m)^2+4x^2\right)} - m^2
\end{array}\right] . \tag{82}$$

$$
K_4 = \left| \begin{array}{c}
-\frac{a\left(x^2+1\right)^2\left(e\left(x^2(e+m)+e-m\right)+x^2+1\right)}{(x^2-1)\left((x^2(e+m)+e-m)^2+4x^2\right)} - \frac{a\left(x^2+1\right)\left((a+1)x^2+a-1\right)}{4x^2} + \frac{\left(x^2(e-m)+e+m\right)\left(x^2(e+m)+e-m\right)}{(x^2-1)^2} - \frac{4x^2}{(x^2-1)^2} \\
0 \\
-\frac{i\left(x^2+1\right)\left(a\left(x^2-1\right)^2\left(x^2+1\right)\left(x^2(e+m)-e+m\right)+8ex^2\left(-\left(x^2(e+m)+e-m\right)^2-4x^2\right)\right)}{2x(x^2-1)^2\left((x^2(e+m)+e-m)^2+4x^2\right)} \\
-\frac{ia\left(x^4-1\right)\left(x^2(e+m)+e-m\right)}{\sqrt{2}x\left((x^2(e+m)+e-m)^2+4x^2\right)} \ ; \\
\\
\frac{ib\left(x^4-1\right)}{\sqrt{2}x(x^2(e+m)+e-m)} \\
\frac{4x^2\left(x^2(e-m)+e+m\right)\left(x^2(e+m)+e-m\right)-b^2\left(x^4-1\right)^2}{4x^2(x^2-1)^2} \\
\frac{b\left(x^4-1\right)}{2\sqrt{2}x^2} \\
-\frac{b\left(x^2+1\right)\left(e\left(x^2+1\right)^3+m\left(x^2-1\right)^3\right)}{4x^2(x^2-1)(x^2(e+m)+e-m)} \ ; \\
\\
\frac{i}{2x}\left(-\frac{a\left(x^2+1\right)^2\left(x^2(e+m)-e+m\right)}{(x^2(e+m)+e-m)^2+4x^2} - \frac{2\left(x^2-1\right)^2}{x^2(e+m)+e-m} + \frac{8e\left(x^2+1\right)x^2}{(x^2-1)^2}\right) \\
\frac{b\left(x^4-1\right)}{2\sqrt{2}x^2} \\
-\frac{\left(a^2+2\right)\left(x^2+1\right)^2}{4x^2} - \frac{a\left(x^2+1\right)^2\left(e\left(x^2(e+m)+e-m\right)+x^2+1\right)}{(x^2-1)\left((x^2(e+m)+e-m)^2+4x^2\right)} + \frac{a-ax^4}{4x^2} + \frac{\left(x^2(e-m)+e+m\right)\left(x^2(e+m)+e-m\right)}{(x^2-1)^2} - \frac{4x^2}{(x^2-1)^2} + 2 \\
\frac{x^2+1}{2\sqrt{2}}\left(\frac{a\left(x^2-1\right)\left(x^2(e+m)+e-m\right)^2}{2x^4(e^2-m^2+2)+x^6(e+m)^2+x^2(e-m)^2} + \frac{4e}{x^2(e+m)+e-m} + \frac{1}{x^2} + 1\right) \ ; \\
\\
\frac{i\left(x^4-1\right)\left(x^2(e+m)-e+m\right)}{\sqrt{2}x\left((x^2(e+m)+e-m)^2+4x^2\right)} \\
-\frac{b\left(x^2+1\right)\left(e\left(x^2+1\right)^3+m\left(x^2-1\right)^3\right)}{4x^2(x^2-1)(x^2(e+m)+e-m)} \\
\frac{x^2+1}{2\sqrt{2}}\left(-\frac{a}{x^2} + a + \frac{4\left(e\left(x^2(e+m)+e-m\right)+x^2+1\right)}{(x^2(e+m)+e-m)^2+4x^2} + \frac{1}{x^2} + 1\right) \\
-\frac{b^2\left(x^2+1\right)^2}{4x^2} + \frac{\left(x^2(e-m)+e+m\right)\left(x^2(e+m)+e-m\right)}{(x^2-1)^2} - \frac{\left(e\left(x^4-1\right)+m\left(x^2-1\right)^2\right)^2}{2x^2\left((x^2(e+m)+e-m)^2+4x^2\right)}
\end{array} \right| . \tag{83}
$$

In addition, we transform derivatives to the new variable, then eqs. (64) and (64))

$$
\left[\frac{d^2}{d\omega^2} + K_1(\omega)\frac{d}{d\omega} + K_3(\omega)\right] F(\omega) = 0 \,, \qquad \left[\frac{d^2}{d\omega^2} + K_2(\omega)\frac{d}{d\omega} + K_4(\omega)\right] G(\omega) = 0
$$

with formulas in mind

$$
\frac{d}{d\omega} = \frac{1+x^2}{2}\frac{d}{dx}, \qquad \frac{d^2}{d\omega^2} = \frac{(1+x^2)^2}{4}\frac{d^2}{dx^2} + 2x(1+x^2)\frac{d}{dx}
$$

take the form (both when $j = 1/2$ and $j \geq 3/2$):

$$
\begin{aligned}
&\left[\frac{d^2}{dx^2} + \frac{2}{1+x^2}\left(K_1(x) + 4x\right)\frac{d}{dx} + \frac{4}{(1+x^2)^2}K_3(x)\right] F = 0 \,, \\
&\left[\frac{d^2}{dx^2} + \frac{2}{1+x^2}\left(K_2(x) + 4x\right)\frac{d}{dx} + \frac{4}{(1+x^2)^2}K_4(x)\right] G = 0 \,.
\end{aligned} \tag{84}
$$

V. SOLUTIONS IN ASYMPTOTIC REGIONS

We will restrict ourselves to the case of minimal $j = 1/2$. First, we find behavior of the blocks $K_i(x)$ in vicinity of the point $x = 0$:

$$K_1(0) = \begin{vmatrix} 0 & -\frac{i}{e+m} & -\frac{i\sqrt{2}}{e+m} \\ -\frac{i}{e+m} & 0 & 0 \\ \frac{i\sqrt{2}}{e+m} & 0 & 0 \end{vmatrix}, \qquad K_2(0) = \begin{vmatrix} 0 & -\frac{i}{e-m} & \frac{i\sqrt{2}}{e-m} \\ -\frac{i}{e-m} & 0 & 0 \\ -\frac{i\sqrt{2}}{e-m} & 0 & 0 \end{vmatrix},$$

$$K_3(0) = \begin{vmatrix} -\frac{1}{2x^2} & -\frac{3i}{2(e+m)x} & -\frac{i}{\sqrt{2}(e+m)x} \\ -\frac{i}{2(e+m)x} & -\frac{1}{x^2} & -\frac{1}{\sqrt{2}x^2} \\ \frac{i}{\sqrt{2}(e+m)x} & -\frac{1}{\sqrt{2}x^2} & -\frac{1}{2x^2} \end{vmatrix}, \tag{85}$$

$$K_4(0) = \begin{vmatrix} \frac{\left(2(e-m)^2+1\right)(e^2-m^2)+2}{2(e-m)^2} & \frac{i}{2(m-e)x} & \frac{i}{\sqrt{2}(e-m)x} \\ \frac{i}{2(e-m)x} & -\frac{1}{2x^2} & \frac{3e^2-4me+m^2+2}{\sqrt{2}(e-m)^2} \\ \frac{i}{\sqrt{2}(e-m)x} & \frac{3e^2-4me+m^2+2}{\sqrt{2}(e-m)^2} & -\frac{1}{2x^2} \end{vmatrix}.$$

Then we find behavior of the blocks $K_i(x)$ in vicinity of the point $x = 1 (r = 1)$:

$$K_1(1) = \begin{vmatrix} \frac{1}{x-1} & -\frac{im}{e^2+1} & 0 \\ -\frac{im}{e^2+1} & \frac{1}{x-1} & 0 \\ 0 & 0 & \frac{1}{x-1} \end{vmatrix}, \qquad K_2(1) = \begin{vmatrix} \frac{1}{x-1} & \frac{im}{e^2+1} & 0 \\ \frac{im}{e^2+1} & \frac{1}{x-1} & 0 \\ 0 & 0 & \frac{1}{x-1} \end{vmatrix},$$

$$K_3(1) = \begin{vmatrix} \frac{e^2-1}{(x-1)^2} & \frac{2ie}{(x-1)^2} & 0 \\ \frac{2ie}{(x-1)^2} & \frac{e^2-1}{(x-1)^2} & -2\sqrt{2} \\ 0 & -2\sqrt{2} & \frac{e^2}{(x-1)^2} \end{vmatrix}, \qquad K_4(1) = \begin{vmatrix} \frac{e^2-1}{(x-1)^2} & \frac{2ie}{(x-1)^2} & 0 \\ \frac{2ie}{(x-1)^2} & \frac{e^2-1}{(x-1)^2} & 2\sqrt{2} \\ 0 & 2\sqrt{2} & \frac{e^2}{(x-1)^2} \end{vmatrix}. \tag{86}$$

We readily find an approximate forms of eqs. (84) in vicinity of two singular points

$$x \to 0, \quad \left[\frac{d^2}{dx^2} + 2K_1(0)\frac{d}{dx} + 4K_3(0)\right] F = 0\,, \quad \left[\frac{d^2}{dx^2} + 2K_2(0)\frac{d}{dx} + 4K_4(0)\right] G = 0\,; \tag{87}$$

$$x \to 1, \left[\frac{d^2}{dx^2} + [(K_1(1) + 4]\frac{d}{dx} + K_3(x)\right] F = 0\,, \quad \left[\frac{d^2}{dx^2} + [K_4(1) + 4]\frac{d}{dx} + K_4(1)\right] G = 0\,. \tag{88}$$

It is evident that in two asymptotic regions $x \to 0$ and $x \to 1$, arising 2-nd order system of equations can be solved. However, hardy this possible in all region of the radial variable $x \in (0, 1)$.

VI. CONCLUSION

In de Sitter space-time, the wave equation for 16-component vector-bispinor related to spin 3/2 particle is presented in the form of three generally covariant equations, one of which can be considered as the main equations, and two additional constraints. This system of equations is specified in de static coordinates of the Sitter model. After separating the variable with the use of operators of energy, total angular momentum, and special reflections, we derive three system of equation with respect to 8 unknown radial function, the case of minimal value j =1/2 is simpler and it includes 6 radial functions. The main task is to find solutions of the main system, first order 8 differential equations with respect to 8 functions. This task is transformed to 2-nd order system for 4 functions, in the case of j=1/2 we have 2-nd order system for 3 functions. We have found their asymptotical behavior near the origin, r=0, and near the event horizon, r=1. Solutions in all region of the radial variable hardly can be found.

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[1] Solutions with Spherical Symmetry of the Wave Equation for a Spin 3/2 Particle / A. V. Ivashkevich, E. M. Ovsiyuk, V. V. Kisel, V. M. Red'kov // P. 168–191 in: Nonlinear Dynamics and Applications: Proceedings of the Twenty six Anniversary Seminar NPCS'2019, Minsk, May 21-24, 2019.

[2] Red'kov V. M. Particle fields in the Riemann space and the Lorentz group. Minsk, Belorusian science Publ., 2009, 496 p.

[3] Red'kov V. M. Tetrad formalism, spherical symmetry and Schrodinger's basis. Minsk, Belorusian science Publ., 2011. 339 p.