# Quantum - classical Wigner-Langevin equation

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The problem consists in an account of Langevin friction and Gaussian noise in known fundamental Wigner-Liouville equation. Langevin friction and Gaussian noise are determined as

$$\mathbf{U_L} = \frac{\hbar \boldsymbol{f}}{\mathbf{2im}} \mathbf{ln}\,\boldsymbol{\Psi}/\boldsymbol{\Psi}^*, \mathbf{U_{rand}}(\mathbf{t}) = -\vec{\mathbf{r}}\vec{\mathbf{F}}_{\mathbf{rand}}(\mathbf{t})$$

$\hbar$ **-** plank constant,

$\boldsymbol{f}$ - friction coefficient,

**Ψ, Ψ*** - wave functions,

**m** - particle mass,

$\vec{\mathbf{F}}_{\mathbf{rand}}$ – noise force,

$\vec{\mathbf{r}}$ – vector coordinate,

**i** – imaginary unit.

In the next transform, it is necessary to transit to new variables

$$\vec{\mathbf{r}} = \overrightarrow{\mathbf{R}} + \frac{1}{2}\hbar\vec{\boldsymbol{\tau}}, \quad \vec{\mathbf{r}}' = \overrightarrow{\mathbf{R}} - \frac{1}{2}\hbar\vec{\boldsymbol{\tau}}$$

and calculate the integral

$$\mathbf{I}_{\boldsymbol{\Sigma}} = \frac{\mathbf{1}}{\mathbf{i}\hbar}\iint\frac{\mathbf{dtd}\vec{\mathbf{p}}'}{(\mathbf{2\pi})^{\mathbf{3}}}\,[\mathbf{U}(\mathbf{1}) + \mathbf{U}(\mathbf{2}) + \mathbf{U}(\mathbf{3})] \bullet$$

$$\bullet\ \mathbf{expi}\ (\vec{\mathbf{p}}' - \vec{\mathbf{p}})\vec{\boldsymbol{\tau}} \bullet \varrho_{\mathbf{w}}(\overrightarrow{\mathbf{R}}, \vec{\mathbf{p}}', \mathbf{t})$$

Here,

$$\mathbf{U}(\mathbf{1}) = \mathbf{U}\left(\overrightarrow{\mathbf{R}} + \frac{\mathbf{1}}{\mathbf{2}}\hbar\vec{\boldsymbol{\tau}}, \mathbf{t}\right) - \mathbf{U}\left(\mathbf{R} - \frac{\hbar\vec{\boldsymbol{\tau}}}{\mathbf{2}}, \mathbf{t}\right)$$

$$\mathbf{U}(\mathbf{2}) = \frac{f}{\mathbf{m}}\left[\mathbf{S}\left(\overrightarrow{\mathbf{R}} + \frac{\mathbf{1}}{\mathbf{2}}\hbar\vec{\boldsymbol{\tau}}, \mathbf{t}\right) - \mathbf{S}\left(\overrightarrow{\mathbf{R}} - \frac{\mathbf{1}}{\mathbf{2}}\hbar\vec{\boldsymbol{\tau}}, \mathbf{t}\right)\right]$$

$$\mathbf{U}(\mathbf{3}) = \mathbf{U}_{\mathbf{rand}}\left(\overrightarrow{\mathbf{R}} + \frac{\mathbf{1}}{\mathbf{2}}\hbar\vec{\boldsymbol{\tau}}, \mathbf{t}\right) - \mathbf{U}_{\mathbf{rand}}\left(\overrightarrow{\mathbf{R}} - \frac{\mathbf{1}}{\mathbf{2}}\hbar\vec{\boldsymbol{\tau}}, \mathbf{t}\right)$$

The quantities **U (1), U (2), U (3)**

are expanded in the Taylor series on $\frac{1}{2}\hbar\vec{\tau}$,

$$\mathbf{U}(\mathbf{1}) = \frac{\partial \mathbf{U}(\vec{\mathbf{R}},\mathbf{t})}{\partial \vec{\mathbf{R}}}\hbar\vec{\tau} + \frac{1}{24}\left(\frac{\partial^3 \mathbf{U}(\vec{\mathbf{R}},\mathbf{t})}{\partial \vec{\mathbf{R}}^3}\right)(\hbar\vec{\tau})^3 + \cdots$$

Now, the integral $\mathbf{I}_{\Sigma}$ on variables $\vec{\tau}$, $\vec{\mathbf{p}}'$ with account of properties delta – function $\boldsymbol{\delta}(\vec{\mathbf{p}}' - \vec{\mathbf{p}})$ permits to visualize the Vigner – Langevin equation as

$$[\frac{\partial}{\partial \mathbf{t}} + \frac{\vec{\mathbf{p}}}{m}\frac{\partial}{\partial \vec{\mathbf{R}}} + \phi(\vec{\mathbf{R}},\mathbf{t})\frac{\partial}{\partial \vec{\mathbf{p}}}]\varrho_w(\vec{\mathbf{R}},\vec{\mathbf{p}},\mathbf{t}) =$$

$$-[\frac{\hbar^2}{24} \bullet \frac{\partial^3}{\partial \vec{\mathbf{R}}^3}[\mathbf{U}(\vec{\mathbf{R}},\mathbf{t}) + \frac{f}{\mathbf{m}}\mathbf{S}(\vec{\mathbf{R}},\mathbf{t})] \bullet \frac{\partial^3}{\partial \vec{\mathbf{p}}^3}]\,\varrho_w(\vec{\mathbf{R}},\vec{\mathbf{p}},\mathbf{t})$$

Where

$$\phi(\vec{\mathbf{R}},\mathbf{t}) = -\frac{\partial \mathbf{U}(\vec{\mathbf{R}},\mathbf{t})}{\partial \vec{\mathbf{R}}} - \frac{f}{\mathbf{m}}\frac{\partial \mathbf{S}(\vec{\mathbf{R}},\mathbf{t})}{\partial \vec{\mathbf{R}}} + \vec{\mathbf{F}}_{\mathbf{rand}}(\mathbf{t}).$$

The additional terms in this equation arise due to friction and noise. In classical limit, there are in combination. In quantum one-half of the equation, it is not such correlation, but term with friction remains.