Is there evidence for dimension-two corrections in QCD via the fits of e^+e^--annihilation data?

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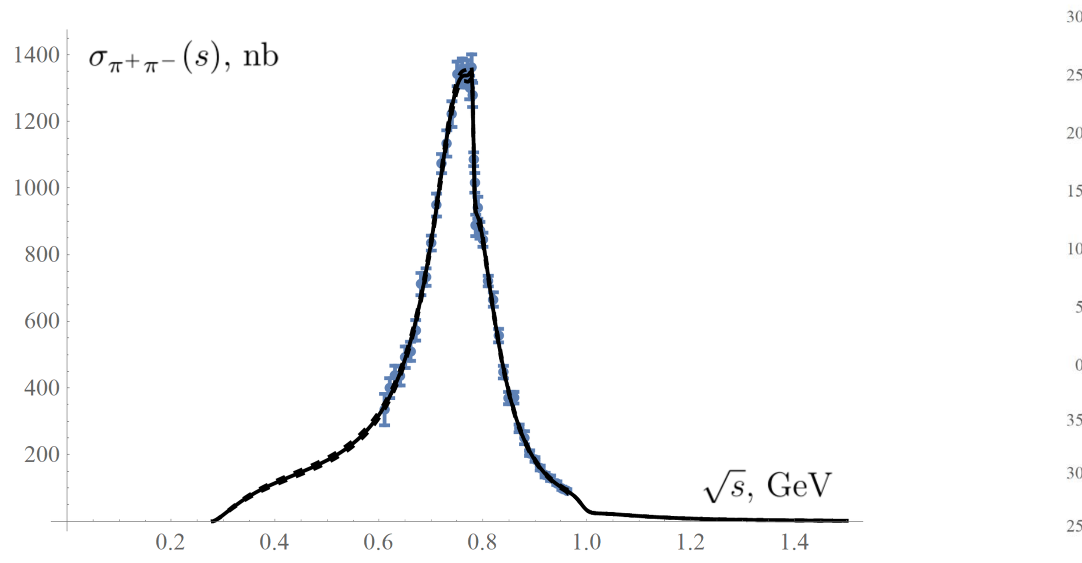
XXVII International Seminar
Nonlinear Phenomena in Complex Systems
The Joint Institute for Power and Nuclear Research – Sosny
May 19-22, 2020 • Minsk • Belarus

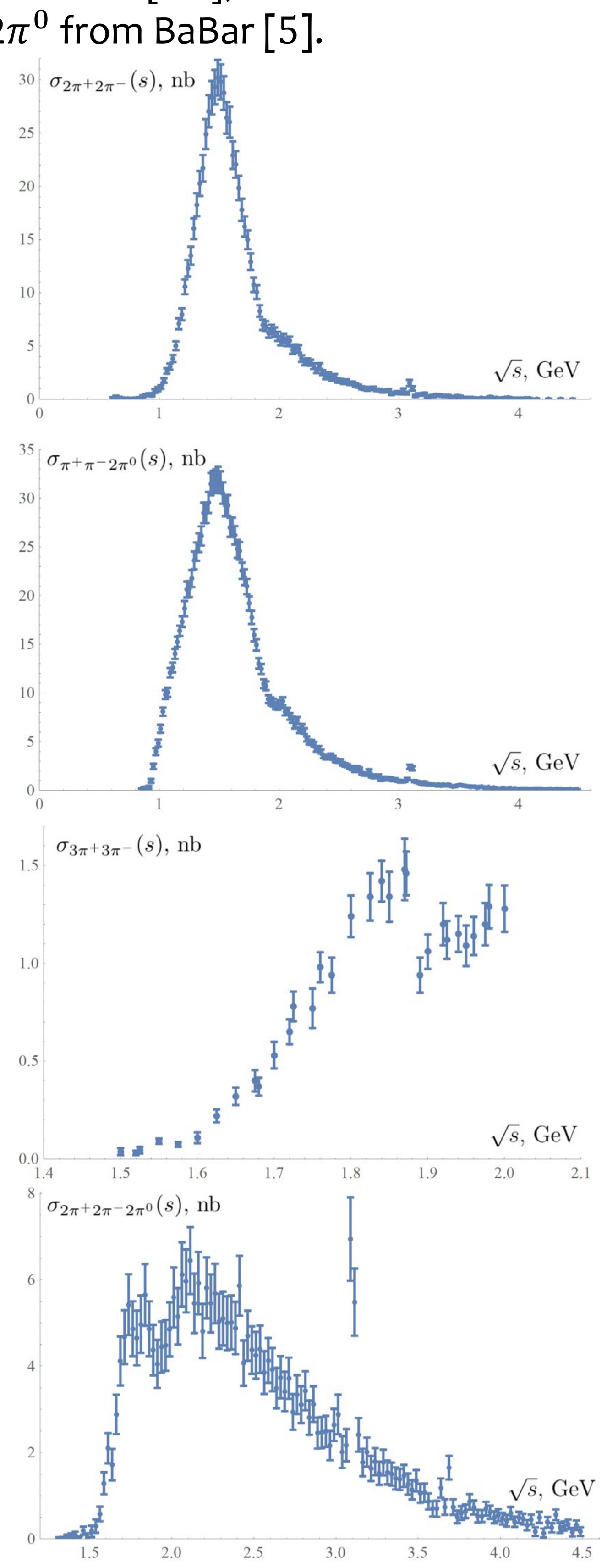
Abstract

We present a re-evaluation of the lowest dimensional condensates of d=2 and 4, as the numerical values of these power correction are not well known, from the high-precision fits of the data on e^+e^- -annihilation into pions: $\pi^+\pi^-$, $2\pi^+2\pi^-$, $\pi^+\pi^-2\pi^0$, $3\pi^+3\pi^-$, and $2\pi^+2\pi^-2\pi^0$. We search if there is an operator with dimension 2 or not and study the difference of the ordinary perturbation theory (PT) and the analytic perturbation theory (APT) results. It is shown that within the framework of the proposed method, the C_2 coefficient of the dimension 2 operator is negative and is closer to zero for the PT. The estimates obtained are more accurate than previously available.

Processing of the experimental data

We consider contemporary experimental data on $e^+e^- \to \pi^+\pi^-$ obtained on the detector CMD-2 [1], on $e^+e^- \to 2\pi^+2\pi^-$ and $e^+e^- \to \pi^+\pi^-2\pi^0$ from BaBar [2,3], on $e^+e^- \to 3\pi^+3\pi^-$ from CMD-3[4] and $e^+e^- \to 2\pi^+2\pi^-2\pi^0$ from BaBar [5].





The data for the process $e^+e^- \to 2\pi^+2\pi^-$ is fitted by Gounaris-Sakurai formula using Breit-Wigner resonances:

$$F^{\rm BW}(s,m_V,\Gamma_V)=\frac{m_V^2(1+d\cdot\Gamma_V/m_V)}{m_V^2-s+f(s,m_V,\Gamma_V)-i\,m_V\Gamma_V(s)},$$

where

$$\Gamma_V(s)=\Gamma_V\left(\frac{k(s)}{k(m_V^2)}\right)^3,\quad k(s)=\frac{\sqrt{s-4m_\pi^2}}{2},$$

$$f(s,m_V,\Gamma_V)=\Gamma_V\frac{m_V^2}{k(m_V^2)^3}\left[k^2(s)(h(s)-h(m_V^2))-(s-m_V^2)k^2(m_V^2)h'(m_V^2)\right],$$

$$h(s)=\frac{2}{\pi}\frac{k(s)}{\sqrt{s}}\ln\left(\frac{\sqrt{s}+2m_\pi}{2m_\pi}\right),\quad h'(m_V^2)=h'(s)\big|_{s=m_V^2},$$

$F^{\rm BW}(0,m_V,\Gamma_V)=1$ automatically.

The full pion form factor with resonances ρ, ω and ρ' takes the form:

$$F_\pi(s,\{\rho,\omega\})=F^{\rm BW}(s,m_\rho,\Gamma_\rho)\frac{1+\alpha_\omega F^{\rm BW}(s,m_\omega,\Gamma_\omega)}{1+\alpha_\omega},$$

where

$$F_\pi(s,\{\rho,\omega,\rho'\})=\frac{F_\pi(s,\{\rho,\omega\})+\alpha_{\rho'}F^{\rm BW}(s,m_{\rho'},\Gamma_{\rho'})}{1+\alpha_{\rho'}}.$$

The fitting results: $\chi^2_{\rm min}=0.9$, $m_\rho=0.774$, Γ_ρ=0.142, m_ω= 0.781, Γ_ω=0.008, α_ω=0.001, $m_{\rho'}$=0.993, $\Gamma_{\rho'}$= 0.046, $\alpha_{\rho'}$= -0.025, d= 0.773.

Theoretical framework: R-ratio and D-function

R-ratio: $R(s)=\dfrac{\sigma_{e^+e^-\to\rm hadrons}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}$, $\sigma_{e^+e^-\to\mu^+\mu^-}(s)=\dfrac{4\pi\alpha_{\rm em}^2}{3s}$.

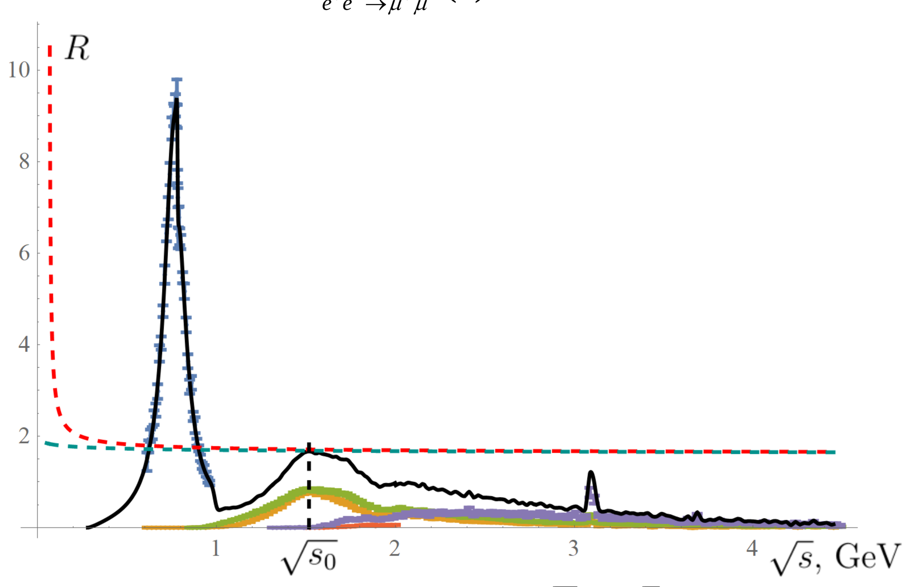


Fig. The full R-ratio in dependence on $\sqrt{s}$ at $\sqrt{s}\le 3$ GeV (black), the experimental data (blue, green, orange, violet and red dots), the theoretical representation $R_{\rm th}$ in the PT (red, dashed) and in the APT (blue, dashed). The continuum threshold is $s_0 \approx 1.52^2$ GeV^2.

Theoretical form: $R_{\rm th}^{\rm PT/APT}(s)=N_c\sum_q e_q^2\left(1+\dfrac{\alpha_s^{\rm PT/APT}(s)}{\pi}\right)$.

Dispersional D-function:

$$D_{\rm exp}(Q^2)=Q^2\int_{4m_\pi^2}^{\infty}\frac{R_{\rm exp\text{-}th}(s)ds}{(s+Q^2)^2}.$$

D-function in OPE framework with using **PT** approach:

$$D_{\rm PT+OPE}(Q^2)=\frac{3}{2}\left(1+\frac{\alpha_s(Q^2)}{\pi}+\sum_{n\ge1}\Gamma(n)\frac{C_{2n}}{Q^{2n}}\right),$$

where $\alpha_s(Q^2)=\dfrac{4\pi}{\beta_0}\dfrac{1}{\ln(Q^2/\Lambda^2)}$,

and **APT** approach:

$$D_{\rm APT+OPE}(Q^2)=\frac{3}{2}\left(1+\frac{\mathcal{A}_s(Q^2)}{\pi}+\sum_{n\ge1}\Gamma(n)\frac{\tilde{C}_{2n}}{Q^{2n}}\right),$$

where

$$\mathcal{A}_s(Q^2)=\frac{4\pi}{\beta_0}\left[\frac{1}{\ln(Q^2/\Lambda_{\rm APT}^2)}-\frac{\Lambda_{\rm APT}^2}{Q^2-\Lambda_{\rm APT}^2}\right].$$

Borel transform (BT) and D-function

BT of D-function: $\hat{B}_{Q^2\to M^2}\left[D_{\rm exp}(Q^2)\right]=\Phi_{\rm exp}(M^2)=\int_{4m_\pi^2}^{\infty}R_{\rm exp\text{-}th}(s)\left(1-\dfrac{s}{M^2}\right)e^{-s/M^2}\dfrac{ds}{M^2}$,

in **PT**: $\hat{B}_{Q^2\to M^2}\left[D_{\rm PT+OPE}(Q^2)\right]=\Phi_{\rm PT+OPE}(M^2)=\dfrac{3}{2}\left(\dfrac{\hat{B}_{Q^2\to M^2}\left[\alpha_s(Q^2)\right]}{\pi}+\dfrac{C_2}{M^2}+\dfrac{C_4}{M^4}+\dfrac{C_6}{M^6}\right)$,

in **APT**: $\hat{B}_{Q^2\to M^2}\left[D_{\rm APT+OPE}(Q^2)\right]=\Phi_{\rm APT+OPE}(M^2)=\dfrac{3}{2}\left(\dfrac{\hat{B}_{Q^2\to M^2}\left[\mathcal{A}_s(Q^2)\right]}{\pi}+\dfrac{\tilde{C}_2}{M^2}+\dfrac{\tilde{C}_4}{M^4}+\dfrac{\tilde{C}_6}{M^6}\right)$.

Equating both forms gives sum rules: $\Phi_{\rm exp}(M^2)=\begin{cases}\Phi_{\rm PT+OPE}(M^2),\\ \Phi_{\rm APT+OPE}(M^2).\end{cases}$

The error of Borelized D-function: $\Delta\Phi_{\rm exp}(M^2)=\int_{4m_\pi^2}^{\infty}\Delta R(s)\left(1-\dfrac{s}{M^2}\right)e^{-s/M^2}\dfrac{ds}{M^2}$.

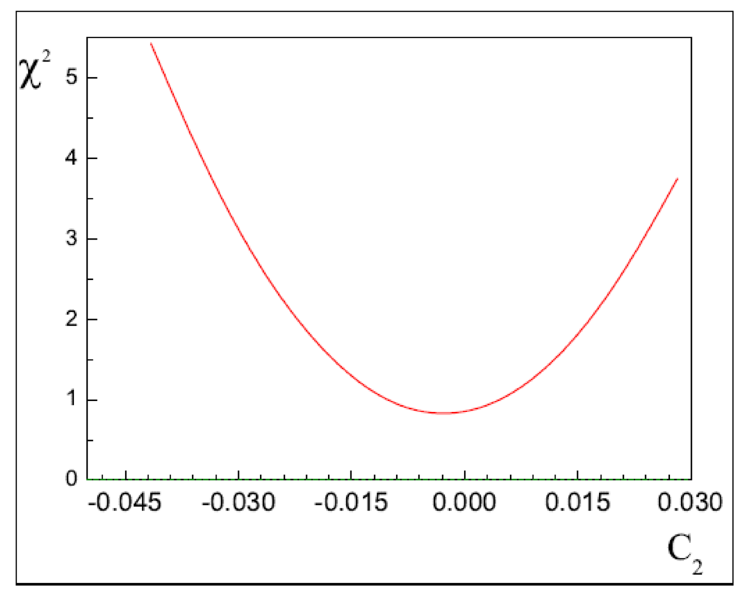
Extracting of condensates

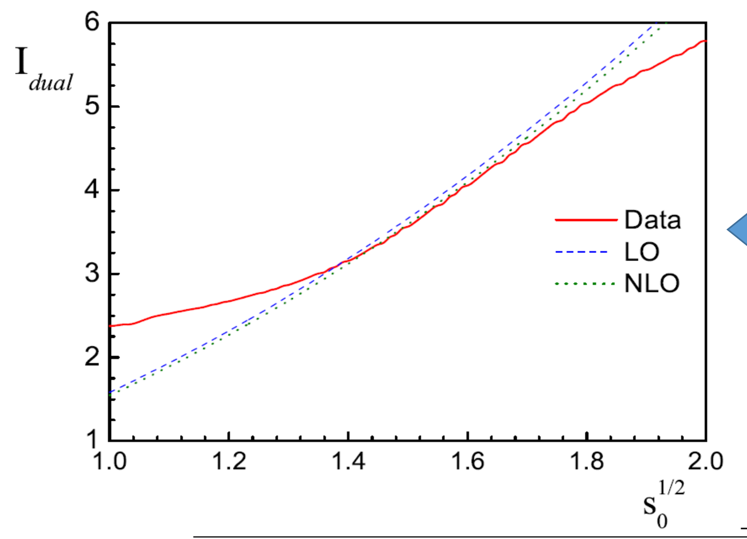
The coefficient $C_6=-\dfrac{448\,\pi^3}{27}\alpha_s\langle\bar{q}q\rangle^2\approx-0.116\ {\rm GeV}^6$ is fixed.

We find C_2 and C_4 by equating both forms of Adler function:

$$\chi^2(C_2,C_4)=\frac{1}{N_{\rm d.f.}}\sum_{n=1}^{N}\frac{\left(\Phi_{\rm exp}(M_n^2)-\Phi_{\rm PT(APT)+OPE}(M_n^2;\{C_2,C_4\})\right)^2}{\Delta\Phi_{\rm exp}(M_n^2)^2}$$

C_4 (GeV^4)	C_2 (GeV^2)	χ^2	$\langle(\alpha_s/\pi)\,GG\rangle$
0.05929	0.019	2.45	0.009011
0.06776	0.011	1.51	0.010298
0.07623	0.003	0.94	0.011586
0.08047	-0.0007	0.78	0.012229
0.08470	-0.0046	0.75	0.012873
0.09317	-0.012	0.94	0.014160
0.10164	-0.020	1.51	0.015448
0.11011	-0.028	2.45	0.016735
0.11858	-0.035	3.78	0.018022
0.12705	-0.043	5.48	0.019309





The value of continuum threshold s_0 we find from the global duality interval condition.

	PT		
Λ_{PT}, GeV	C_2, GeV^2	C_4, GeV^4	$\dfrac{\langle\alpha_s GG\rangle}{\pi}$, GeV^4
0.250	-0.003 ∓ 0.015	0.084 ± 0.012	0.013 ± 0.002
0.300	-0.032 ∓ 0.016	0.093 ± 0.013	0.014 ± 0.002
0.350	-0.062 ∓ 0.015	0.100 ± 0.012	0.015 ± 0.002

	APT		
Λ_{APT}, GeV	$\tilde{C}_2$, GeV^2	$\tilde{C}_4$, GeV^4	$\dfrac{\langle\alpha_s GG\rangle}{\pi}$, GeV^4
0.277	0.022 ± 0.015	0.088 ± 0.012	0.013 ± 0.002
0.342	0.005 ± 0.016	0.099 ± 0.012	0.015 ± 0.002
0.412	-0.011 ∓ 0.016	0.110 ± 0.012	0.017 ± 0.002

Conclusion

The best fitting PT results:

$$C_2=-0.003\mp0.015\ {\rm GeV}^2$$

$$\langle\alpha_s GG\rangle/\pi=3/(2\pi^2)C_4=0.013\pm0.002{\rm GeV}^4$$

The negative C_2 may be partially responsible for short strings effect. Comparison of the obtained values with our previous analysis [9]: $C_2=-0.047\mp0.052\ {\rm GeV}^2$

$\langle\alpha_s GG\rangle/\pi=3/(2\pi^2)C_4=0.018\pm0.008\ {\rm GeV}^4$.

demonstrates the strong dependence on the set of experimental e^+e^--annihilation data and their accuracy.

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Thank you for your attention!