Informational Approach in Quantum Mechanics and its Applications to Quantum Chromodynamics

V. I. Kuvshinov* and E. G. Bagashov†

Joint Institute for Power and Nuclear Research - Sosny,
National Academy of Sciences, P.O. box 119, Minsk, 220109, Belarus

The theory of quantum information is a well-developed field with many non-trivial results that currently find practical applications in quantum computing and other areas. We present some general ideas from this field regarding processes of measurement and then we apply these results to a different field of study: strong interaction physics. It is shown that here certain characteristic phenomena may be associated with the informational processes, such as the decoherence of colour state due to the interaction of colour charges with the environment of colour fields.

PACS numbers: 12.38.Aw, 05.45.Mt
Keywords: quantum information, density matrix, stochastic vacuum, quantum environment, quarks, quantum chromodynamics, colour confinement

I. INTRODUCTION

Quantum information theory – as well as its practical applications e.g. in quantum optics – has become one of the main avenues of research in physics in the recent decades. It has many potential and promising applications, such as the construction of quantum computers and the establishment of quantum cryptography. On the other hand, some have argued that this theory is the only way of resolving the fundamental problems of quantum mechanics itself [1].

One of the fundamental problems regarding the quantum theory is the problem of measurement [2]. In particular, the superposition state which certain quantum systems might find themselves in, is not observed experimentally, since it doesn't have macroscopic analogs and is a purely quantum phenomenon [3]. This gives rise to the problem of state vector collapse (also known as reduction of wave function) during the measurement – the unexplained disappearance of superposition state in the measurement results [4]. A well known illustration of this is Schrödinger's cat paradox [5]. Before the measurement the cat is either dead, alive, or dead and alive simultaneously (superposition).

There are different interpretations of quantum mechanics. The most popular ones are [6, 7] (in descending order):

- Copenhagen;
- Many-worlds;
- Informational.

Each one of these interpretations treats the problem of state vector collapse differently. There is another approach (also recognized in [6, 7]), which is usually not considered to be a separate interpretation, yet is directly connected to the problem of measurement and state vector collapse: decoherence [8].

---

*Electronic address: v.kuvshinov@sosny.bas-net.by
†Electronic address: bagashov@sosny.bas-net.by

In this approach the state vector collapse is explained as a consequence of the interaction of the system with the environment: the part of information about the superposition state is "smeared" over the system and the environment, and if the latter is large enough, it is no longer possible (in practical sense) to extract it (although theoretically it is still possible – see Section III for discussion). It is still present in the overall system (including the quantum system and the environment), but not observed. Characteristic time of decoherence $\sim \frac{\hbar^2}{m}$ [9] (where m is the mass of the system, including the environment), i.e. it happens rather quickly, and the faster, the larger m is.

In this approach all the equations of quantum mechanics are valid for all the scales, i.e. there is no boundary between the quantum and classical worlds, and classical laws are just the approximations of quantum ones, valid for large systems, where superpositions quickly decay. Effectively, it eliminates the problem of wave function collapse (it is an apparent effect, resulting from the large size of classical objects) and gives an adequate answer to the role of the observer (the existence of consciousness is not required, decoherence is objective) [9].

II. APPLICATION OF DECOHERENCE TO QCD

Quark model, which was successfully used in quantum chromodynamics to describe the observed statistics of hadrons, introduces an additional quantum number (colour) in order to be compatible with the Pauli exclusion principle. However, the particles with colour charge (quarks and gluons) are not directly observed in experiment – the effect which is referred to as confinement of colour charge.

The confinement is the situation when the physical spectrum is devoid of particles (fields) that are present in the fundamental Lagrangian [10]. In case of QCD this means that quarks, gluons and other colour objects cannot exist as separate asymptotic states. Confinement of quarks and gluons represents a serious theoretical challenge due to the non-perturbative dynamics of underlying fields (caused by the large coupling constant at low transferred momenta), which is problematic in terms of finding an analytical description [10].

In this work we shall describe the use of the method of vacuum correlators to analyze the confinement and other non-perturbative dynamics of quarks [11].

An informational treatment of the problem might be found in [12], where the interaction of an arbitrary colour superposition with the stochastic QCD vacuum as environment is considered. Let us consider colour basis of vectors $|1\rangle$, $|2\rangle$, $|3\rangle$; then we would have a superposition vector

$$|\phi_{in}\rangle = \alpha|1\rangle + \beta|2\rangle + \gamma|3\rangle, \tag{1}$$

where the weight factors are normalized as

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1. \tag{2}$$

The corresponding initial density matrix might be written as

$$\hat{\rho}_{in} = |\phi_{in}\rangle\langle\phi_{in}|, \tag{3}$$

or, in the basis of vectors $|1\rangle$, $|2\rangle$, $|3\rangle$:

$$\hat{\rho}_{in} = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \alpha^*\beta & |\beta|^2 & \beta\gamma^* \\ \alpha^*\gamma & \beta^*\gamma & |\gamma|^2 \end{pmatrix}. \tag{4}$$

In the following derivation we should also take into account the fact that the SU(N) group generators $\hat{t}_a$ (with the unit operator $\hat{I}$) might serve as a basis in the space of Hermitian matrices, and therefore we might decompose the density matrix as [13]

$$\hat{\rho} = N_c^{-1}\hat{I} + \rho_1^a\hat{t}_a, \tag{5}$$

where $N_c = 3$ is the number of colours.

Omitting some intermediate steps, we obtain the resulting density matrix (after the interaction with the stochastic vacuum) by averaging the resulting expression with respect to all the implementations of the vacuum

$$\hat{\rho}_f =< \hat{\rho} >= N_c^{-1}\hat{I} + (\hat{\rho}_{in} - N_c^{-1}\hat{I})W_{adj}, \tag{6}$$

where $< ... >$ denotes the averaging over vacuum implementations, and W_{adj} is the Wilson loop in the adjoint representation [14]. By definition $W = \mathrm{Tr}\left(\mathcal{P}\exp i\int dx^\mu \hat{A}_\mu\right)$. In the confinement region we have [14]

$$W_{adj} = \exp(-\sigma_{adj}RT), \tag{7}$$

where RT is the area of the 1×1-dimensional rectangular loop and σ_{adj} is the QCD string tension in the adjoint representation. Therefore,

$$\hat{\rho}_f(RT \to \infty) = N_c^{-1}\hat{I}. \tag{8}$$

Thus the interaction of initial superposition (3) with the stochastic vacuum leads to the emergence of a state with equal probabilities for different colours [12] (prehadrons). In the initial basis this might be written as

$$\begin{pmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \alpha^*\beta & |\beta|^2 & \beta\gamma^* \\ \alpha^*\gamma & \beta^*\gamma & |\gamma|^2 \end{pmatrix} \to \begin{pmatrix} N_c^{-1} & 0 & 0 \\ 0 & N_c^{-1} & 0 \\ 0 & 0 & N_c^{-1} \end{pmatrix} \tag{9}$$

Thus, decoherence leads to the appearance of a mixed state without the non-diagonal elements of density matrix and equal probabilities for all colours.

In the non-asymptotic cases we might use series expansion of the exponent (7):

$$\hat{\rho}_f = N_c^{-1}\hat{I} + (\hat{\rho}_{in} - N_c^{-1}\hat{I})\exp(-\sigma RT). \tag{10}$$

In the case $0 < \sigma RT \ll 1$:

$$\exp(-\sigma RT) = \sum_{i=0}^{\infty}\frac{(-\sigma RT)^i}{i!} \approx 1 - \sigma RT, \tag{11}$$

so (10) becomes

$$\hat{\rho}_f = \hat{\rho}_{in} - (\hat{\rho}_{in} - N_c^{-1}\hat{I})\sigma RT, \tag{12}$$

or, alternatively,

$$\hat{\rho}_f = \hat{\rho}_{in}(1 - \sigma RT) + \sigma RT N_c^{-1}\hat{I}. \tag{13}$$

The generalization of this approach to the case of many-particle systems has been considered in [15, 16], and it has been shown that the resulting density matrix is

$$\hat{\rho} = N_c^{-N_p}\hat{I} + (\hat{\rho}_{in} - N_c^{-N_p}\hat{I})W_{adj}^{N_p}, \tag{14}$$

where N_p is the number of particles. Correspondingly,

$$\hat{\rho}(RT \to \infty) = N_c^{-N_p}\hat{I}. \tag{15}$$

III. DISCUSSION

The nature of quantum measurement and the colour confinement is common: both might be interpreted as quantum decoherence that arises during the interaction of the system with the environment, in our case represented by stochastic QCD vacuum.

Let us consider certain theorems known in quantum information theory: namely, the no-cloning and no-hiding theorems.

The no-cloning theorem [17] states that it is impossible to create an identical copy of an arbitrary unknown quantum state. It might also be generalized for mixed quantum states, in which case it is called the no-broadcasting theorem. In particular, the theorem implies that it is impossible to measure a quantum state without disturbing it. Which means that measurement erases initial quantum information from the measured system. So in our case it is possible to say that the interaction of a quark with the stochastic vacuum of QCD leads to the loss of information on the colour state of quark itself.

The no-hiding theorem [18] states that if information is lost from the original system, it moves from the system to the environment. One important consequence of this is the fact that the quantum information is never lost entirely. During the measurement it redistributes to the environmental degrees of freedom. So in case of the interaction of a quark with the stochastic vacuum the information on the initial colour of the quark is being transferred to the stochastic vacuum, yet it is lost for the original object (quark).

And, finally, it should be mentioned that some recent studies have suggested to use entanglement entropy as an observable in deep inelastic scattering experiments [19], which later was extended to p–p processes [20]. The obtained results seem to indicate that quantum entanglement might be present at sub-nucleonic scales, which is another indication that quantum information processes provide a good alternative description of the strong interaction phenomena.

IV. CONCLUSIONS

With the help of model of QCD stochastic vacuum we have shown that the interaction of a pure colour state with the QCD stochastic vacuum (treated as an environment) leads to the appearance of a mixed state. The bigger is the interval (represented formally through the Wilson loop), the more information on the colour state of the initial quark is lost (and the closer the final state is to the fully mixed one), which corresponds to confinement of colour charges. Evolution of multiparticle systems might be considered, as well as various specific types of quantum states.

Confinement of colour charges might be considered as one of the consequences of decoherence and no-cloning theorem, while no-hiding theorem provides an opportunity to evaluate the amount of lost information.

[1] C. A. Fuchs. Quantum Mechanics as Quantum Information (and only a little more). arXiv:quant-ph/0205039 (2002).

[2] Landau L. D. Course of Theoretical Physics / Landau L. D., Lifshitz E. M. Vol. 3: Quantum Mechanics: Non-Relativistic Theory, Nauka, Moscow, 1989.

[3] Dirac P. A. M. The Principles of Quantum Mechanics, Clarendon Press, Oxford, 1947.

[4] W. Heisenberg. Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik, Zeitschrift für Physik. no. 43, P. 172–198.

[5] E. Schrödinger. Die gegenwärtige Situation in der Quantenmechanik. Die Naturwissenschaften, Vol. 23, no. 48, P. 807–812 (1935).

[6] M. Schlosshauer, J. Kofler, A. Zeilinger. A snapshot of foundational attitudes toward quantum mechanics. Die NaturwissenschaftenStudies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics, Vol. 44, no. 3, P. 222–230 (2013).

[7] S. Sivasundaram, K. H. Nielsen. Surveying the Attitudes of Physicists Concerning Foundational Issues of Quantum Mechanics. arXiv:1612.00676 (2016).

[8] H.–D. Zeh. On the Interpretation of Measurement in Quantum Theory // Foundations of Physics. 1970. Vol. 1, no. 1. P. 69–76.

[9] W. H. Zurek. Decoherence, einselection, and the quantum origins of the classical. Reviews of Modern Physics, vol. 75, 2003, P. 715–775.

[10] Y. A. Simonov. The Confinement. Uspekhi Fizicheskih Nauk. **4** (1996) (in Russian).

[11] A. D. Giacomo, H. Dosch, V. I. Shevchenko, and Y. A. Simonov, Physics Reports **372**, no. 4, 319–368 (2002).

[12] V. I. Kuvshinov, E. G. Bagashov. Evolution of Colour Superposition in the Stochastic QCD Vacuum // Nonlinear Phenomena in Complex Systems. 2013. V. 16, no. 3. P. 242–246.

[13] E. Brüning, H. Mäkelä, A. Messina, F. Petruccione. Parametrizations of density matrices. Journal of Modern Optics, **59**, no. 1, 1–20 (2012).

[14] K. G. Wilson. Confinement of quarks. Phys. Rev. D 10, 2445–2459 (1974).

[15] Kuvshinov V. I. Confinement of Colour States in A Stochastic Vacuum of Quantum Chromodynamics/ Kuvshinov V. I. and. Bagashov E. G. Theoretical and Mathematical Physics // 2015, Vol.184, no. 3, P.1304–1310.

[16] V. Kuvshinov. Decoherence of Quantum States in QCD Vacuum / Kuvshinov V., Bagashov E. // Physics of Particles and Nuclei, 2017, Vol. 48, no. 5, P. 834–835.

[17] J. Park. The concept of transition in quantum mechanics // Foundations of Physics, no. 1, P. 23–33 (1970).

[18] S. L. Braunstein and A. K. Pati. Quantum Information Cannot be Completely Hidden in Correlations: Implications for Black Hole Information Paradox. Phys. Rev. Lett. 98, 080502 (2007).

[19] D. E. Kharzeev, E. M. Levin. Deep inelastic scattering as a probe of entanglement, Phys. Rev. D95, no. 11, 114008 (2017).

[20] Z. Tu, D. Kharzeev, T. Ullrich. The EPR paradox and quantum entanglement at sub-nucleonic scales. arXiv:1904.11974 [hep-ph] (2019).