SURFACE PLASMON-POLARITONS AT THE INTERFACE OF MAGNETOELECTRIC HYPERBOLIC METAMATERIAL

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Abstract

In this paper, we investigated the features of plasmon-polaritons excited at the interface of a magnetoelectric hyperbolic metamaterial and a dielectric for the case when the optical axis is arbitrary oriented under the normal to the boundary. Expressions are obtained for the complex electric and magnetic vectors as well as for the decay constants of the fields on both sides of the interface. The possibility is shown and the conditions are determined for localization of plasmon-polariton at the boundary of metamaterial of different types. It is shown that the wave vector of plasmon-polariton has the component oriented perpendicular to the boundary. It is established that for metamaterials of different types changing the orientation of the optical axis one can realize the conditions when the phase velocity of plasmon-polariton is directed from the boundary inside a metamaterial or a dielectric.

1. Introduction

Last decade hyperbolic metamaterials (HMM) have attracted significant scientific interest due to their extraordinary properties and great prospects for the sub-wavelength imaging [1, 2], the control of spontaneous emission [3], the thermal radiative heat transfer [4]. Recently the hyperbolic metamaterials with tilted optical axes are synthesized. Meanwhile, it is shown that they exhibit asymmetry properties for waves propagating upward and downward with respect to slab interfaces [5, 6]. This is potentially prospective for creation of new types of optical devices, for example, an absorber [5], meta-waveguides and sensors [6], a wave expander and a source shifter [7]. As a rule, authors consider non-magnetic hyperbolic metamaterials (including HMM with tilted optical axes). In this paper we investigate the influence of magnetoelectric properties of HMM with arbitrary optical axis on the conditions of existence of surface plasmon-polaritons.

2. Surface plasmon-polariton at the boundary of isotropic medium and magnetoelectric hyperbolic metamaterial with arbitrary oriented optical axis

2.1. Features of plasmon-polaritons at the boundary of isotropic medium and magnetoelectric hyperbolic metamaterial with arbitrary oriented optical axis

Let us consider the boundary of isotropic dielectric with permittivity ε_1 and permeability $\mu_1 \cong 1$ and magnetoelectric hyperbolic metamaterial. In effective medium theory when the thickness of each layer is sufficiently small, i.e. $|k_d d_d| << 1, |k_m d_m| << 1$ where k_d, k_m are the wave numbers of dielectric and metallic layers, respectively, this structure can be considered as anisotropic effective medium described by uniaxial tensors

$$\varepsilon_2 = diag\{\varepsilon_\perp, \varepsilon_\perp, \varepsilon_\parallel\} = \varepsilon_\perp + \delta \mathbf{c} \otimes \mathbf{c}, \quad \mu_2 = diag\{\mu_\perp, \mu_\perp, \mu_\parallel\} = \mu_\perp + \chi \mathbf{c} \otimes \mathbf{c}. \qquad (1)$$

Here we consider the simplest case when electric and magnetic axes coincide, $\delta = \varepsilon_\parallel - \varepsilon_\perp, \chi = \mu_\parallel - \mu_\perp$, $\varepsilon_\parallel(\mu_\parallel)$ and $\varepsilon_\perp(\mu_\perp)$ are the longitudinal (along the Z' axis) and transverse (in the plane orthogonal to Z') main permittivities (permeabilities), $\mathbf{c}$ is the unit vector along the HMM optical axis, and the symbol $\otimes$ denotes the dyadic product of the vectors ($\mathbf{c} \otimes \mathbf{c} = c_i c_k$). For simplicity of our consideration we support that $\mathrm{Im}(\varepsilon_\perp) \approx 0$, $\mathrm{Im}(\varepsilon_\parallel) \approx 0$. As calculation shows this assumption is correct for spectral regions where parameters $\varepsilon_\perp$ ($\mu_\perp$) and $\varepsilon_\parallel$ ($\mu_\parallel$) significantly differ from zero.

Let us the angle between the optical axis of the metamaterial and the normal to the HMM boundary (Z axis) is θ (Fig.1). For definition we suggest that the optical axis is disposed between the Z and X axes of chosen coordinate system.

Now we consider *p*-polarized surface waves propagating along the *X* axis in the plane separated the isotropic dielectric and hyperbolic metamaterial. We use the coordinate system *XYZ* (see Fig.1) in which the effective permittivity (permeability) tensor ε' (μ'), characterizing the hyperbolic metamaterial, is represented in the form:

$$\varepsilon' = U\varepsilon\tilde{U}, \mu' = U\mu\tilde{U} \qquad (2)$$

Here U is the transformation matrix which permits to transfer from the coordinate system related to the HMM main dielectric (magnetic) axes to the system of coordinate *XYZ*:

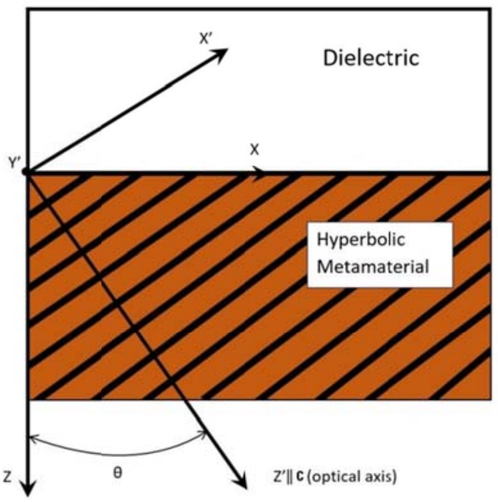


Figure 1: Schematic view of the hyperbolic metamaterial bordered with the dielectric. The Z′ axis is parallel to the **c** vector.

$$U = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}, \qquad (3)$$

the symbol "tilde" denotes the transposition. As follows from Eqs.(1) and (2)

$$\begin{aligned} &\varepsilon' = U\varepsilon\widetilde{U} = \varepsilon_{xx}\mathbf{e}_x \otimes \mathbf{e}_x + \varepsilon_{\perp}\mathbf{e}_y \otimes \mathbf{e}_y + \varepsilon_{zz}\mathbf{e}_z \otimes \mathbf{e}_z + \\ &+ \varepsilon_{xz}(\mathbf{e}_x \otimes \mathbf{e}_z + \mathbf{e}_z \otimes \mathbf{e}_x), \\ &\mu' = U\mu\widetilde{U} = \mu_{xx}\mathbf{e}_x \otimes \mathbf{e}_x + \mu_{\perp}\mathbf{e}_y \otimes \mathbf{e}_y + \mu_{zz}\mathbf{e}_z \otimes \mathbf{e}_z + \\ &+ \mu_{xz}(\mathbf{e}_x \otimes \mathbf{e}_z + \mathbf{e}_z \otimes \mathbf{e}_x). \end{aligned} \qquad (4)$$

Here

$$\begin{aligned} \varepsilon_{xx} &= \varepsilon_{\perp}\cos^2\theta + \varepsilon_{\parallel}\sin^2\theta, \\ \varepsilon_{zz} &= \varepsilon_{\perp}\sin^2\theta + \varepsilon_{\parallel}\cos^2\theta, \\ \varepsilon_{xz} &= (\varepsilon_{\parallel} - \varepsilon_{\perp})\sin\theta\cos\theta, \end{aligned} \qquad (5)$$

$$\begin{aligned} \mu_{xx} &= \mu_{\perp}\cos^2\theta + \mu_{\parallel}\sin^2\theta, \\ \mu_{zz} &= \mu_{\perp}\sin^2\theta + \mu_{\parallel}\cos^2\theta, \\ \mu_{xz} &= (\mu_{\parallel} - \mu_{\perp})\sin\theta\cos\theta, \end{aligned} \qquad (6)$$

$\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are the unit vectors of used coordinate system.

Now we represent the vectors of the field inside the dielectric (*d*) and metamaterial (*m*) in the form

$$\mathbf{F} = \begin{cases} \mathbf{F}^d \exp(\kappa_d z + iqx - i\omega t), & z < 0 \\ \mathbf{F}^m \exp(-\kappa_m z + iqx - i\omega t), & z > 0 \end{cases}, \qquad (7)$$

where $\mathbf{F}$ denotes electric $\mathbf{E} = (E_x, 0, E_z)$ or magnetic $\mathbf{H} = (0, H_y, 0)$ vector, $\kappa_{d,m} > 0$ and *q* are the decay constants and longitudinal wave number of the surface wave (plasmon-polariton), respectively, $\kappa_d^2 = q^2 - k_0^2\varepsilon_1$, $k_0 = \omega / c$, ω is the cyclic frequency of electromagnetic wave, *c* is the light velocity in vacuum. Taking into account Eq. (7) from the Maxwell equations one can obtain the expressions for electric $\mathbf{E}^d, \mathbf{E}^m$ and magnetic $\mathbf{H}^d, \mathbf{H}^m$ vectors inside the dielectric and metamaterial:

$$\begin{aligned} \mathbf{E}^d &= -\frac{A_0}{\varepsilon_1}\left(i\frac{\kappa_d}{k_0}\mathbf{e}_x + \frac{q}{k_0}\mathbf{e}_z\right)e^{iqx + \kappa_d z}, \\ \mathbf{H}^d &= A_0\mathbf{e}_y e^{iqx + \kappa_d z}. \end{aligned} \qquad (8)$$

$$\begin{aligned} \mathbf{E}^m &= \frac{A_0}{sk_0}[(i\kappa_m\varepsilon_{zz} + q\varepsilon_{xz})\mathbf{e}_x - \\ &-(q\varepsilon_{xx} + i\kappa_m\varepsilon_{xz})\mathbf{e}_z]e^{iqx - \kappa_m z}, \\ \mathbf{H}^m &= A_0\mathbf{e}_y e^{iqx - \kappa_m z}. \end{aligned} \qquad (9)$$

Here $s = \varepsilon_{xx}\varepsilon_{zz} - \varepsilon_{xz}^2$, A_0 is the amplitude, and the phase factor $\exp(-i\omega t)$ is omitted. Substituting Eqs. (8), (9) into the Maxwell equation $rot\mathbf{E} = -(1/c)\mu\partial\mathbf{H}/\partial t$ we find:

$$\kappa_m^2 = aq^2 - bk_0^2 + 2iqd\kappa_m. \qquad (10)$$

Here

$$a = \varepsilon_{xx} / \varepsilon_{zz}, b = s\mu_{\perp} / \varepsilon_{zz}, d = \varepsilon_{xz} / \varepsilon_{zz}.$$

If *d* parameter is small the decay constant κ_m is represented in the form:

$$\kappa_m = \kappa_{mo} + i\kappa_{m1} = \sqrt{aq^2 - bk_0^2} + idq. \qquad (11)$$

It follows from Eq. (11) that the decay constant κ_m is complex. Its real part depends on both dielectric and magnetic properties of metamaterial. But nonzero propagation constant appears for the case when optical axis of HMM is not orthogonal to its interface. Thereby, the wave of special type exists on the metamaterial boundary. Its phase velocity is directed under the γ angle towards the HMM boundary:

$$\tan\gamma = -d = -\frac{(\varepsilon_{\parallel} - \varepsilon_{\perp})\sin\theta\cos\theta}{\varepsilon_{\perp}\sin^2\theta + \varepsilon_{\parallel}\cos^2\theta}. \qquad (12)$$

As follows from Eq. (12), two different cases of orientation of the phase velocity of this wave can be realized (Fig.2). As seen from Fig.2, for these two cases the planes of equal phase and equal amplitudes are not collinear. When $\varepsilon_{\perp} > 0, \varepsilon_{\parallel} < 0$ we have

$$d = \frac{\sin 2\theta}{2\left(\dfrac{|\varepsilon_{\parallel}|}{|\varepsilon_{\parallel}| + \varepsilon_{\perp}} - \sin^2\theta\right)}. \qquad (13)$$

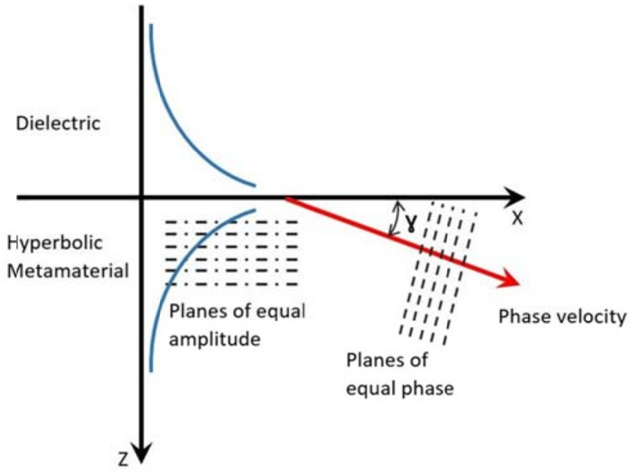
When $\varepsilon_{\perp} < 0, \varepsilon_{\parallel} > 0$ parameter *d* is represented in the form

$$d = \frac{\sin 2\theta}{2\left(\frac{\varepsilon_{\parallel}}{\varepsilon_{\parallel} + |\varepsilon_{\perp}|} - \sin^2\theta\right)}. \qquad (14)$$

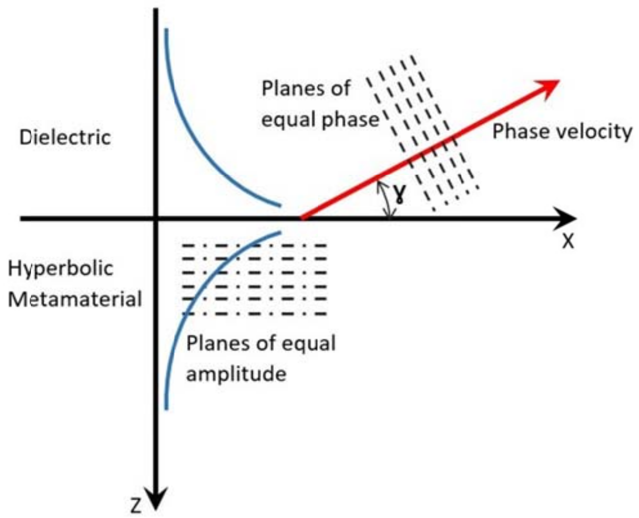
If $0 < \theta < \pi/2$ the parameter d is positive (and the phase velocity is directed towards the dielectric) for the following range of angles:

$$\sin\theta < \sqrt{\frac{|\varepsilon_{\parallel}|}{|\varepsilon_{\parallel}| + \varepsilon_{\perp}}} \quad (\text{for } \varepsilon_{\perp} > 0, \varepsilon_{\parallel} < 0), \qquad (15a)$$

$$\sin\theta < \sqrt{\frac{\varepsilon_{\parallel}}{\varepsilon_{\parallel} + |\varepsilon_{\perp}|}} \quad (\varepsilon_{\perp} < 0, \varepsilon_{\parallel} > 0). \qquad (15b)$$



(a)



(b)

Figure 2: Orientation of the phase velocity of surface wave at the boundary of hyperbolic metamaterial for the case of negative (a) and positive (b) values of the parameter *d*.

As is seen from Eqs.(13),(14), the function $d(\theta)$ has extremum if the following condition is fulfilled:

$$(1+\cos 2\theta)^2 + 2\cos 2\theta\left(\frac{|\varepsilon_{\parallel}|}{|\varepsilon_{\parallel}| + |\varepsilon_{\perp}|} - 1 - \cos^2\theta\right) = 0. \qquad (16)$$

From the boundary conditions for electric $\mathbf{E}^d, \mathbf{E}^m$ and magnetic $\mathbf{H}^d, \mathbf{H}^m$ vectors (see Eqs. (8), (9)) it follows the dispersion equation for plasmon-polariton:

$$\frac{1}{s}(i\kappa_{mo}\varepsilon_{zz} + q\varepsilon_{xz}) = -\frac{1}{\varepsilon_1}(i\kappa_d). \qquad (17)$$

For $\theta \cong 0$ or $\theta \cong \pi/2$ the simple ratio for real part of the longitudinal wave number of the plasmon-polariton follows from Eq. (17):

$$q = k_0\sqrt{\frac{\varepsilon_1\varepsilon_{zz}(\varepsilon_{xx} - \varepsilon_1\mu_{\perp})}{\varepsilon_{xx}\varepsilon_{zz} - \varepsilon_1^2}} = k_0\sqrt{\varepsilon_{eff}}. \qquad (18)$$

2.2. The condition of the TM polarized plasmon-polariton localization at the boundary of magnetoelectric hyperbolic metamaterial

Now we will analyze the possibility of localization of the electromagnetic wave on the interface of an isotropic medium and a magnetoelectric hyperbolic metamaterial. Due to the exponentially decaying of the amplitude of this field when moving off the boundary, the wave number q has to be larger than modules of wave vectors of the waves propagating inside the dielectric and metamaterial. Thereby,

$$\varepsilon_{eff} > 0,\ \varepsilon_{eff} > \varepsilon_1,\ \kappa_{mo} > 0. \qquad (19)$$

Conditions (19) are fulfilled if following irregularities take place:

$$\varepsilon_{xx}\varepsilon_{zz} < 0, \mu_{\perp} > 0,\ \varepsilon_{zz}\mu_{\perp} > \varepsilon_1. \qquad (20)$$

Using Eq.(5) we obtain that this condition is fulfilled for the angles

$$(1-|\Delta|)/2 < \sin^2\theta < (1+|\Delta|)/2, \qquad (21)$$

where

$$\Delta = \begin{cases} \dfrac{\varepsilon_{\parallel} - |\varepsilon_{\perp}|}{\varepsilon_{\parallel} + |\varepsilon_{\perp}|} & \text{if} \quad \varepsilon_{\perp} < 0, \varepsilon_{\parallel} > 0 \\ \dfrac{|\varepsilon_{\parallel}| - \varepsilon_{\perp}}{|\varepsilon_{\parallel}| + \varepsilon_{\perp}} & \text{if} \quad \varepsilon_{\perp} > 0, \varepsilon_{\parallel} < 0 \end{cases}. \qquad (22)$$

Simplifying ratios (22) and using (20) and (5) we have different possible cases.

1. $\varepsilon_{\perp} < 0, \varepsilon_{\parallel} > 0,\ \varepsilon_{\parallel} > |\varepsilon_{\perp}|$

$$\frac{|\varepsilon_{\perp}|}{\varepsilon_{\parallel} + |\varepsilon_{\perp}|} < \sin^2\theta < \frac{\varepsilon_{\parallel}}{\varepsilon_{\parallel} + |\varepsilon_{\perp}|}, \quad \sin^2\theta < \frac{\varepsilon_{\parallel} - \varepsilon_1/\mu_{\perp}}{\varepsilon_{\parallel} + |\varepsilon_{\perp}|}. \qquad (23a)$$

2. $\varepsilon_{\perp} < 0, \varepsilon_{\parallel} > 0,\ \varepsilon_{\parallel} < |\varepsilon_{\perp}|$

$$\frac{\varepsilon_{\|}}{\varepsilon_{\|}+|\varepsilon_{\perp}|}<\sin^2\theta<\frac{|\varepsilon_{\perp}|}{\varepsilon_{\|}+|\varepsilon_{\perp}|},\quad \sin^2\theta<\frac{\varepsilon_{\|}-\varepsilon_1/\mu_{\perp}}{\varepsilon_{\|}+|\varepsilon_{\perp}|}. \tag{23b}$$

3. $\varepsilon_{\perp}>0, \varepsilon_{\|}<0, |\varepsilon_{\|}|>\varepsilon_{\perp}$

$$\frac{\varepsilon_{\perp}}{|\varepsilon_{\|}|+\varepsilon_{\perp}}<\sin^2\theta<\frac{|\varepsilon_{\|}|}{|\varepsilon_{\|}|+\varepsilon_{\perp}},\quad \sin^2\theta>\frac{|\varepsilon_{\|}|+\varepsilon_1/\mu_{\perp}}{|\varepsilon_{\|}|+\varepsilon_{\perp}}. \tag{23c}$$

4. $\varepsilon_{\perp}>0, \varepsilon_{\|}<0, |\varepsilon_{\|}|<\varepsilon_{\perp}$

$$\frac{|\varepsilon_{\|}|}{|\varepsilon_{\|}|+\varepsilon_{\perp}}<\sin^2\theta<\frac{\varepsilon_{\perp}}{|\varepsilon_{\|}|+\varepsilon_{\perp}},\quad \sin^2\theta>\frac{|\varepsilon_{\|}|+\varepsilon_1/\mu_{\perp}}{|\varepsilon_{\|}|+\varepsilon_{\perp}}. \tag{23d}$$

As follows from Eqs. (23), there are the ranges of the angles θ for which plasmon-polaritons existed at the boundary of metamaterial and dielectric are localized. Meanwhile, these ranges depend on dielectric and magnetic properties of HMM.

As an example we consider the possibility of existence of localized TM polarized plasmon-polaritons in magnetoelectric metamaterial on the basis of nanostructure the unit cell of which is formed by the nanolayers of silver (Ag) and dielectric TiO_2 [8]. As follows from the date in Ref. [8], at the wavelength λ=600 nm this structure can be considered as effective medium described by uniaxial tensors of permittivity and permeability. Meanwhile, $\varepsilon_{\perp}\approx-2; \varepsilon_{\|}\approx 10; \mu_{\perp}\approx 2.5; \mu_{\|}\approx 1.5.$ Then the case 1 is realized, and TM polarized localized plasmon-polaritons existed at the boundary of metamaterial and optical glass BK7, for example, under the condition $2.3^o<\theta<59^o$. It should be noted that the presence of magnetic properties of HMM reduces the angle interval for which localized plasmon-polaritons are maintained at the boundary.

3. Conclusions

Thus, in this paper we investigated the features of plasmon-polaritons excited at the interface of a magnetoelectric hyperbolic metamaterial and a dielectric for the case when the optical axis is arbitrary oriented under the normal to the boundary. Expressions are obtained for the complex electric and magnetic vectors as well as for the decay constants of the fields on both sides of the interface. It is shown that the wave vector of plasmon-polariton has the component oriented perpendicular to the boundary, and this component is independent of permeability but determined by dielectric properties of metamaterial.

It is established the conditions of exciting the localized TM plasmon-polaritons at the boundary of magnetoelectric HMM. It is found the regions of orientation of the optical axis for which these plasmons remain localized.

For illustration of the obtained results the numerical modeling is carried out of the plasmon-polariton for the case of hyperbolic metamaterial formed on the basis of the multilayered TiO_2/Ag nanostructure.

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