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**Threshold resummation S factor for a system
of two relativistic spinor quarks with equal masses**

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Plan

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1. Introduction

By describing quark-antiquark systems near their production threshold $s = 4m^2$ we can not cut off the perturbative series even if the QCD coupling constant α_s is small [T. Appelquist, H. D. Politzer, *Phys. Rev. Lett.* **34**, 43 (1975); *Phys. Rev. D* **12**, 1404 (1975)]. The reason is that, in the threshold region, one has to construct a perturbation expansion in terms of α_s/v , which is a singular quantity, where

$$v = \sqrt{1 - \frac{4m^2}{s}} \quad (1)$$

is the relative velocity of fermions (or quarks) in the c.m. frame above threshold, \sqrt{s} being the total c.m. energy of interacting particles and m there is their masses (we use the system of units where $\hbar = c = 1$). In order to obtain a meaningful result, threshold singularities of the form $(\alpha_s/v)^n$ should be summed.

In the nonrelativistic case for the Coulomb interaction

$$V(r) = -\frac{\alpha_s}{r} \quad (2)$$

these threshold singularities in the form $(\alpha_s/v)^n$ can be explicitly summarized by the known S -factor Gamov–Sommerfeld–Sakharov [G. Gamov, *Zeit. Phys.* **51**, 204 (1928); see also L. I. Schiff, *Quantum Mechanics*, 142 (1955); A. Sommerfeld, *Atombau und Spektrallinien*, **II** (1939); A. D. Sakharov, *Zh. Eksp. Teor. Fiz.* **18**, 631 (1948)]

$$S_{\text{nr}} = \frac{X_{\text{nr}}}{1 - \exp(-X_{\text{nr}})}, \quad X_{\text{nr}} = \frac{\pi\alpha_s}{v_{\text{nr}}}, \quad (3)$$

which is related to the normalized wave function of the continuous spectrum at the origin by $|\psi(0)|^2$, where $2v_{\text{nr}}$ is the relative velocity of two nonrelativistic particles.

In relativistic theory, the nonrelativistic expression in (3) for two spinless particles of equal mass should be modified.

For the first time the relativization of the S -factor (3) in QCD in the case of $m_1 = m_2$ was executed in [V. S. Fadin, V. A. Khoze, *Yad. Fiz.* **48**, 487 (1988); V. S. Fadin, V. A. Khoze, A. D. Martin, and A. Chapovsky, *Phys. Rev. D* **52**, 1377 (1995)] and it consisted in $v_{\text{nr}} \rightarrow v$. This factor was used for the description of effects close to the threshold of pair production in the processes $e^+e^- \rightarrow t\bar{t}$ and $e^+e^- \rightarrow W^+W^-$. Just same form of the S -factor but with $v_{\text{nr}} \rightarrow v_{\text{rel}} = 2v/(1 + v^2)$ for the interaction of $m_1 = m_2$ was later suggested in [A. H. Hoang, *Phys. Rev. D* **56**, 7276 (1997)]. Another form of the relativistic generalization of the S -factor also in the case of $m_1 = m_2$ was obtained in [J. H. Yoon and C. Y. Wong, *Phys. Rev. C* **61**, 044905 (2000); *J. Phys. G: Nucl. Part. Phys.* **31**, 149 (2005)]. The relativistic S -factor for $m_1 \neq m_2$ was presented in [A. B. Arbuzov, *Nuov. Cim. A* **107**, 1263 (1994)]. This factor was derived within the framework of relativistic quantum mechanics on the basis of the Schrödinger equation.

The new method to the relativization of the S -factor in the case of $m_1 = m_2$ was developed by Milton and Solovtsov in [K. A. Milton, I. L. Solovtsov, *Mod. Phys. Lett. A* **16**, 2213 (2001)]. Their the method is based on the relativistic quasipotential (RQP) approach proposed by [A. A. Logunov, A. N. Tavkhelidze, *Nuov. Cim. A* **29**, 380 (1963)], in the form suggested by [V. G. Kadyshevsky, *Nucl. Phys. B* **6**, 125 (1968)]. This approach is a new step to application of the quasipotential approach in QCD and it gives the following expression for the relativistic S -factor:

$$S(\chi) = \frac{X(\chi)}{1 - \exp[-X(\chi)]}, \quad X(\chi) = \frac{\pi\alpha_s}{\sinh \chi} = \pi\alpha_s \sqrt{1 - v^2}/v, \quad (4)$$

where χ is the rapidity related to the total c. m. energy of particles, \sqrt{s} by

$$2m \cosh \chi = \sqrt{s}. \quad (5)$$

Their the method the possibility of transformation of RQP equation from momentum space into relativistic configurational representation (RCR) in the case of two particles of equal masses has been used also [V. G. Kadyshevsky, R. M. Mir-Kasimov, N. B. Skachkov, *Nuov. Cim. A* **55**, 233 (1968)] and, it is important, that they has used the potential (2) which possesses the QCD-like behaviour [V. I. Savrin, N. B. Skachkov, *Lett. Nuov. Cim.* **29**, 363 (1980)]. However, it was shown in [O. P. Solovtsova and Yu. D. Chernichenko, *Phys. At. Nucl.* **73**, 1612 (2010); *Theor. Math. Phys.* **166**, 194 (2011)] that the relativistic limits of the S -factors in [A. H. Hoang, *Phys. Rev. D* **56**, 7276 (1997); J. H. Yoon and C. Y. Wong, *Phys. Rev. C* **61**, 044905 (2000); *J. Phys. G: Nucl. Part. Phys.* **31**, 149 (2005); A. B. Arbuzov, *Nuov. Cim. A* **107**, 1263 (1994)] differ substantially from the relativistic limit ($v \rightarrow 1$) of the S -factor in (4) which tends to unity in the relativistic limit.

Applications of the factor (4) for describing some hadronic processes can be found in papers [I. L. Solovtsov, O. P. Solovtsova, *Nonlinear Phenom. in Complex Syst.* **5**, 51 (2002); *Actual problems of particle physics: Proc. of Int. School-Seminar, JINR, Dubna I*, 312 (2002); *Nuclear Science and Safety in Europe*. Eds. T. Čechhák e. a., **161**, (2006); K. A. Milton, I. L. Solovtsov, O. P. Solovtsova, *Phys. Rev. D* **64**, 016005 (2001); **65**, 076009 (2002); *Mod. Phys. Lett. A* **16**, 2213 (2001); **21**, 1355 (2006)].

Recently, the relativistic *S*-factor (4) has been applied in paper [K. A. Milton, *Proc. of the Int. Seminar Denoted to the Memory of I. L. Solovtsov, Dubna, 15-18 Jan. 2008. JINR. P. 82*] to reanalyze the mass limits obtained for magnetic monopoles which might have been produced at the Fermilab Tevatron.

The resummation factors appears too in the parametrization of the imaginary part of the corresponding quark current correlators, in the Drell ratio $R(s)$ [K. Adel, F. J. Yndurain, *Phys. Rev. D* **52**, 6577 (1995)], which in the two-particle approximation can be approximated in terms of the Bethe–Salpeter (BS) amplitude of two charged particles $\chi_{\text{BS}}(x)$ at $x = 0$ [R. Barbieri, P. Christillin, E. Remiddi, *Phys. Rev. D* **8**, 2266 (1973)]. The nonrelativistic replacement of this amplitude by the wave function, which obeys the Schrödinger equation with the Coulomb potential (2), leads to formula (3) with a substitution $\alpha_s \rightarrow 4\alpha_s/3$ for QCD.

The possibility of using the RQP approach for our task is based on the fact that the BS amplitude, which parameterizes the physical quantity $R(s)$ and is taken at $x = 0$ and hence at the relative time $\tau = 0$, can be expressed in the case of the interaction of two relativistic particles of equal masses m through the wave RQP function in the momentum space, $\Psi_q(\mathbf{p})$, and in the configuration representation, $\psi_q(\mathbf{r})$, as

$$\chi_{\text{BS}}(x = 0) = \frac{1}{(2\pi)^3} \int d\Omega_{\mathbf{p}} \Psi_q(\mathbf{p}) = \psi_q(\mathbf{r}) \Big|_{r=i\lambda}, \quad (6)$$

where $\lambda = 1/m$ is the Compton wavelengs particle of mass m , $d\Omega_{\mathbf{p}} = (m d\mathbf{p})/p_0$ is the relativistic three-dimensional volume element in the Lobachevsky space realized on the hyperboloid $p_0^2 - \mathbf{p}^2 = m^2$.

The purpose of this paper is to generalize the method proposed in [K. A. Milton, I. L. Solovtsov, *Mod. Phys. Lett. A* **16**, 2213 (2001); O. P. Solovtsova and Yu. D. Chernichenko, *Phys. At. Nucl.* **73**, 1612 (2010); *Theor. Math. Phys.* **166**, 194 (2011)] for the case a composite system formed by two relativistic quarks having equal masses and a spin of $1/2$ and interacting via a Coulomb-like chromodynamic potential (2). The pseudoscalar, vector, and pseudovector cases are considered, and the behavior of the *S-factor* is analyzed in the nonrelativistic, relativistic, and ultrarelativistic cases. This investigation be based on the RQP approach to quantum field theory [A. A. Logunov, A. N. Tavkhelidze, *Nuov. Cim. A* **29**, 380 (1963)], in the form suggested by [V. G. Kadyshevsky, *Nucl. Phys. B* **6**, 125 (1968)] via a transition from the momentum representation in Lobachevsky space to the three-dimensional RCR introduced in [V. G. Kadyshevsky, R. M. Mir-Kasimov, N. B. Skachkov, *Nuov. Cim. A* **55**, 233 (1968)] for a composite system of two relativistic equal-mass particles.

2. Coulomb wave function: the case of two relativistic spinor particles of equal masses

The present analysis be based on the fully covariant two-particle three-dimensional RQP equation in an integral form for the case of two relativistic spinor equal-mass particles. In the configuration representation, this equation for the radial RQP wave function of relative orbital angular momentum $\ell = 0$ has the form [Yu. D. Chernichenko, *Phys. At. Nucl.* **80**, 707 (2017)]

$$\int_0^{\infty} d\chi' (\cosh \chi - \cosh \chi') \sin \rho \chi' \int_0^{\infty} d\rho' \sin(\rho' \chi') \varphi_0(\rho', \chi) = \quad (7)$$

$$= V(\rho) \int_0^{\infty} d\chi' \hat{A}(\cosh \chi') \sin \rho \chi' \int_0^{\infty} d\rho' \sin(\rho' \chi') \varphi_0(\rho', \chi),$$

the potential $V(\rho)$, $\rho = r/\lambda$, is local in the sense of Lobachevskys

geometry, and the operator \hat{A} is given by

$$\hat{A} \left(\frac{\hat{H}_0}{2m} \right) = \frac{1}{4} \left[a \left(\frac{\hat{H}_0}{2m} \right)^2 + b \right], \quad (8)$$

$$a = \begin{cases} 1 & \text{for } \hat{O} = \gamma_5, \text{ (pseudoscalar);} \\ \frac{1}{2} & \text{for } \hat{O} = \gamma_\mu, \text{ (vector);} \\ -\frac{1}{2} & \text{for } \hat{O} = \gamma_5 \gamma_\mu, \text{ (pseudovector);} \end{cases}$$

$$b = \begin{cases} 0 & \text{for } \hat{O} = \gamma_5, \text{ (pseudoscalar);} \\ \frac{1}{4} & \text{for } \hat{O} = \gamma_\mu, \text{ (vector);} \\ \frac{3}{4} & \text{for } \hat{O} = \gamma_5 \gamma_\mu, \text{ (pseudovector).} \end{cases}$$

A solution of RQP equation (7) with the potential (2) is sought in the form

$$\varphi_0(\rho, \chi) = \int_{\alpha_-}^{\alpha_+} d\zeta e^{i\rho\zeta} R_0(\zeta, \chi), \quad (9)$$

where integration is performed in a complex plane along a contour between the end points $\alpha_{\pm} = -R \pm i\varepsilon$, $R \rightarrow +\infty$, $\varepsilon \rightarrow +0$, the points $\pm\chi + 2\pi ni$ ($n = 0, \pm 1, \dots$) are the branch points of the function $R_0(\zeta, \chi)$, and the contour of integration must not intersect cuts which we take from $-\infty + 2\pi ni$ to $\pm\chi + 2\pi ni$ (Fig. 1), that is, as in [K. A. Milton, I. L. Solovtsov, *Mod. Phys. Lett. A* **16**, 2213 (2001); O. P. Solovtsova and Yu. D. Chernichenko, *Phys. At. Nucl.* **73**, 1612 (2010); *Theor. Math. Phys.* **166**, 194 (2011); Yu. D. Chernichenko, *Phys. At. Nucl.* **80**, 707 (2017); I. L. Solovtsov, Yu. D. Chernichenko, *Int. Sem. on Contemporary Probl. of Elem. Part. Phys., Dedicated to the Memory of I. L. Solovtsov, Dubna, Jan. 17-18, 2008*. Proc.–Dubna: JINR, 2008, D4-2008-65, p. 73.]

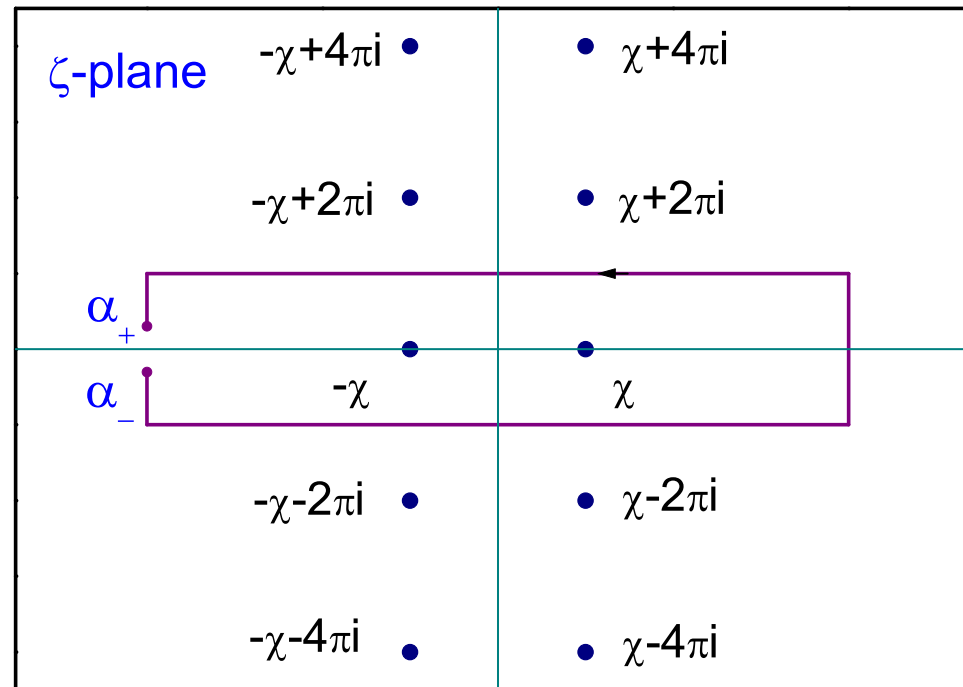


Fig. 1: Contour of integration in Eq. (9) and singularities of the function (12) in the complex ζ -plane.

Substituting Eqs. (2) and (9) into Eq. (7), taking into account that

$$\frac{1}{i\pi} \int_0^{\infty} d\rho' \sin(\rho' \chi') e^{i\rho' \zeta} = \frac{1}{i\pi} \frac{\chi'}{\chi'^2 - \zeta^2},$$

and performing integration by parts, we arrive to the equation

$$\frac{d}{d\zeta} [(\cosh \chi - \cosh \zeta) R_0(\zeta, \chi)] - i\tilde{\alpha}_s \hat{A}(\cosh \zeta) R_0(\zeta, \chi) = 0, \tilde{\alpha}_s = m\alpha_s, \quad (10)$$

with the boundary condition

$$e^{i\rho\zeta} (\cosh \chi - \cosh \zeta) R_0(\zeta, \chi) \Big|_{\zeta=\alpha_-}^{\zeta=\alpha_+} = 0. \quad (11)$$

As a result the solution of Eq. (10) with the boundary condition (11) is

$$R_0(\zeta, \chi) = \tag{12}$$

$$= C_0(\chi) \frac{\exp \left[-\frac{i\tilde{\alpha}_s a}{4} \sinh \zeta + (1 - i\tilde{\rho})\zeta + iB\chi \right]}{(e^\zeta - e^\chi)^2} \left[\frac{e^\zeta - e^{-\chi}}{e^\zeta - e^\chi} \right]^{-1+iB},$$

where $C_0(\chi)$ is an arbitrary function of χ , the parameters a, b and $\tilde{\alpha}_s$ are defined in Eqs. (8) and (10), and the parameters $\tilde{\rho}$ and B are given by

$$\tilde{\rho} = \frac{\tilde{\alpha}_s a \cosh \chi}{4}, \quad B = \frac{\tilde{\alpha}_s (a \cosh^2 \chi + b)}{4 \sinh \chi}. \tag{13}$$

At $\chi = i\kappa$, the parameter B is related to the quantization condition [Yu. D. Chernichenko, Phys. At. Nucl. **80**, 707 (2017)]

$$\frac{\tilde{\alpha}_s (a \cos^2 \kappa + b)}{4 \sin \kappa} = n, \quad n = 1, 2, \dots, \quad 0 < \kappa < \pi/2. \tag{14}$$

In the case when the interaction vanishes, $\alpha_s \rightarrow 0$, the solution $\varphi_0(\rho, \chi)$ should reproduce the known free wave function

$$\lim_{\alpha_s \rightarrow 0} \varphi(\rho, \chi) \xrightarrow{\rho \rightarrow \infty} \frac{\sin(\rho\chi)}{\sinh \chi}. \quad (15)$$

Substituting the solution (12) into (9) and performing ζ -integration in the complex plane along a contour with end points α_{\pm} [K. A. Milton, I. L. Solovtsov, *Mod. Phys. Lett. A* **16**, 2213 (2001); O. P. Solovtsova and Yu. D. Chernichenko, *Phys. At. Nucl.* **73**, 1612 (2010); *Theor. Math. Phys.* **166**, 194 (2011); Yu. D. Chernichenko, *Phys. At. Nucl.* **80**, 707 (2017); I. L. Solovtsov, Yu. D. Chernichenko, *Int. Sem. on Contemporary Probl. of Elem. Part. Phys., Dedicated to the Memory of I. L. Solovtsov, Dubna, Jan. 17-18, 2008. Proc.–Dubna: JINR, 2008, D4-2008-65, p. 73.*] we obtain the resulting solution which does not contain the i -periodic constant:

$$\varphi_0(\rho, \chi) = 2 C_0(\chi) e^{iB\chi} \sinh [\pi(\rho - \tilde{\rho})] \times \quad (16)$$

$$\times \int_{-\infty}^{\infty} dx \frac{\exp \left[\frac{i\tilde{\alpha}_s a}{4} \sinh x + (1 + i(\rho - \tilde{\rho})) x \right]}{(e^x + e^{-x})^2} \left[\frac{e^x + e^{-x}}{e^x + e^x} \right]^{-1+iB}.$$

One can readily find that the normalization factor $C_0(\chi)$ in the solution given by (16) is real-valued. In addition, we see that, apart from the oscillating factor $\exp[i\tilde{\alpha}_s a \sinh x/4]$, the solution in (16) coincides in form with the solution in the spinless case, and at $a = 0$ and $b = 2$, the former reduces to the latter [O. P. Solovtsova and Yu. D. Chernichenko, *Phys. At. Nucl.* **73**, 1612 (2010); *Theor. Math. Phys.* **166**, 194 (2011)]. Taking the foregoing into consideration, we will set the oscillating factor $\exp[i\tilde{\alpha}_s a \sinh x/4]$ to unity in the exact solution in (16). Such approximation not only does not break characteristic to symmetries of the solution in (16), but also allows

to present expression for the RQP radial wave function for the *s*-wave state in terms of a hypergeometric function [O. P. Solovtsova and Yu. D. Chernichenko, *Phys. At. Nucl.* **73**, 1612 (2010); *Theor. Math. Phys.* **166**, 194 (2011)]

$$\begin{aligned} \varphi_0(\rho, \chi) &= & (17) \\ &= 2\pi C_0(\chi) e^{iB\chi - \chi + i(\rho - \tilde{\rho})\chi} (\rho - \tilde{\rho}) F(1 - iB, 1 - i(\rho - \tilde{\rho}); 2; 1 - e^{-2\chi}), \end{aligned}$$

where the parameters $\tilde{\rho}$ and B were defined in Eqs. (13), and the real-valued normalization factor $2\pi C_0(\chi)$ gives by the expression

$$|2\pi C_0(\chi)|^2 = e^{\pi B} |\Gamma(1 - iB)|^2, \quad (18)$$

which one can derive from the boundary condition in (15) and the asymptotic expression

$$\begin{aligned} \varphi_0(\rho, \chi)|_{\rho \gg 1} &\approx \frac{2\pi C_0(\chi) e^{-\pi B/2}}{\sinh \chi |\Gamma(1 - iB)|} \sin \{ (\rho - \tilde{\rho})\chi + \\ &+ B \ln [2(\rho - \tilde{\rho}) \sinh \chi] + \arg \Gamma(1 - iB) \}. \end{aligned}$$

3. Threshold S -factor for a system of two relativistic spinor quarks of equal masses

We define the threshold S -factor in the spinor case as

$$S_{\text{RQP,S}}(\chi) = \lim_{\rho \rightarrow i} \left| e^{-\pi\tilde{\rho}/2} \Gamma(1 + i\tilde{\rho}) \frac{\varphi_0(\rho, \chi)}{\rho} \right|^2, \quad (19)$$

where not only does the additional factor $\exp(-\pi\tilde{\rho}/2)\Gamma(1 + i\tilde{\rho})$ lead to the correct relativistic limit for $\chi \rightarrow +\infty$, which is equal to unity, but it also ensures a transition to the spinless case at $a = 0$ and $b = 2$. Thus, we see that, in the spinor case, the function

$$\psi_0(\rho, \chi) = e^{-\pi\tilde{\rho}/2} \Gamma(1 + i\tilde{\rho}) \varphi_0(\rho, \chi)$$

is a physical wave function for the Coulomb interaction (2).

Since the BS amplitude $\chi_{\text{BS}}(x=0)$ is related to the RQP wave function $\psi_q(\mathbf{r})$ by Eq. (6), the following expression [Yu. D. Chernichenko, *Phys. At. Nucl.* **82**, 158 (2019)] for the relativistic threshold S -factor in the case of a composite system formed by two relativistic spinor particles of equal masses can be obtained with the aid of relations (17)–(19)

$$S_{\text{RQP,S}}(\chi) = \quad (20)$$

$$= \frac{X_{\text{RQP,S}}(\chi)}{1 - \exp[-X_{\text{RQP,S}}(\chi)]} e^{-\pi\tilde{\rho}} |\Gamma(2 + i\tilde{\rho}) F(1 + iB, -i\tilde{\rho}; 2; 1 - e^{-2\chi})|^2.$$

Here, the quantity

$$X_{\text{RQP,S}}(\chi) = 2\pi B = \frac{\pi\tilde{\alpha}_s(a \cosh^2 \chi + b)}{2 \sinh \chi} = \frac{\pi\tilde{\alpha}_s(a + b - bv^2)}{2v\sqrt{1 - v^2}}. \quad (21)$$

It is noteworthy that, at $a = 0$ and $b = 2$, the relativistic threshold resummation S -factor (20) goes over to the spinless S -factor (4), which, in the nonrelativistic limit ($v \ll 1$), reproduces the well-known nonrelativistic result (3) [O. P. Solovtsova and Yu. D. Chernichenko, *Phys. At. Nucl.* **73**, 1612 (2010); *Theor. Math. Phys.* **166**, 194 (2011)].

Let us study the behavior of the relativistic threshold resummation S -factor (20) in the nonrelativistic ($\chi \rightarrow +0$), relativistic ($\chi \rightarrow +\infty$), and ultrarelativistic limits. In the nonrelativistic limit, we have the expression

$$S_{\text{RQP,S}}(\chi)|_{\chi \rightarrow +0} \approx \frac{\pi \tilde{\alpha}_s (a+b)/2 \sinh \chi}{1 - \exp[-\pi \tilde{\alpha}_s (a+b)/2 \sinh \chi]} \frac{\pi \tilde{\alpha}_s a/2}{\exp(\pi \tilde{\alpha}_s a/2) - 1} \left(1 + \frac{\tilde{\alpha}_s^2 a^2}{16}\right),$$

which, as was mentioned above, reduces to the spinless S -factor (4) at $a = 0$ and $b = 2$.

In the relativistic limit ($v \rightarrow 1$), we have

$$S_{\text{RQP,S}}(\chi)|_{\chi \rightarrow +\infty} \approx \frac{2\pi(B - \tilde{\rho})}{1 - \exp[-2\pi(B - \tilde{\rho})]} \xrightarrow{\chi \rightarrow +\infty} 1 + 0.$$

This expression is valid at all values of the spin parameters a and b in (8).

As was proven in [W. Lucha, F. F. Schöberl, *Phys. Rev. Lett.* **64**, 2733 (1990); *Phys. Lett. B* **387**, 573 (1996)], the spectrum of bound states vanishes in the ultrarelativistic limit for $m \rightarrow 0$, since the particle mass is the only dimensional parameter. This feature reflects a substantial difference between potential models and quantum field theory, where there arises an additional dimensional parameter. In addition, we can conclude that the S -factor corresponding to the continuous spectrum should tend to unity for $m \rightarrow 0$.

Thus, we have established the dependence of the relativistic threshold resummation *S*-factor (20) on the spin parameters a and b . The above analysis of its behavior in the nonrelativistic ($v \ll 1$), relativistic ($v \rightarrow 1$), and ultrarelativistic ($m \rightarrow 0$) cases has shown that this resummation factor reproduces both the well-known nonrelativistic limit in the spinless case, where $a = 0$ and $b = 2$, and the expected relativistic and ultrarelativistic limits for all of the three cases: the pseudoscalar, vector, and pseudovector ones.

5. Conclusions

In the present study, a new threshold resummation S -factor (20) has been obtained for a composite system of two relativistic spinor quarks having equal masses and interacting via a Coulomb-like chromodynamic potential. The pseudoscalar, vector, and pseudovector cases have been considered. For this purpose, a fully covariant Hamiltonian formulation of the quasipotential approach in quantum field theory has been implemented via a transition to the three-dimensional relativistic configuration representation for the case of a composite system formed by two relativistic spinor particles of equal masses. The dependence of the relativistic threshold resummation S -factor (20) on the spin parameters a and b has been found. It has been shown that, at $a = 0$ and $b = 2$, the relativistic S -factor (20) reduces to the spinless S -factor in (4).

The present analysis of the behavior of the S -factor (20) in the nonrelativistic ($v \ll 1$), relativistic ($v \rightarrow 1$), and ultrarelativistic ($m \rightarrow 0$) limits has shown that it reproduces both the known nonrelativistic limit in the spinless case of $a = 0$ and $b = 2$ and the expected relativistic and ultrarelativistic limits for all of three cases: the pseudoscalar, vector, and pseudovector ones.

Since expression (20) has been obtained here for the S -factor within a fully covariant method, it can be expected that the relativistic character of interacting particles and spin effects have been comprehensively taken into account.

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