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Plane Quazicrystals and Four Colors Problem

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Recently authors the plane quasicrystals simple dynamic model has constructed defined by the dynamic system acting upon the interval $[0,1]$ (for instance) as follows

$$\left(\prod_{1\leqslant n\leqslant 5} \cos(\mu_n(x,y)+B+D\Phi)+1\right):2 \mapsto \Phi$$

where $\mu_n(x,y) \stackrel{def}{=} x\sin(2\pi n) - y\cos(2\pi n)$ and B is real number.

The dynamic system action result do not contain any additional effects. There are subsist only five families of straight lines defined by respective equations

$$\mu_n(x,y)+B=C, \quad n\in\overline{1,5},$$

such that for all real numbers $A\in[0,1]$ and C there exists positive integer N, such that

$$\Phi_N(C+DA)=A, \quad \Phi_0=\mathrm{id}.$$

Theorem 1 *The straight lines intersection points are stable and unstable three-separatrix fixed or periodic points.*

Simple geometric considerations deliver the following statement

Theorem 2 *There are only three stable and unstable separatries for every fixed or periodic points.*

It is means that there are not exist fixed or periodic antisaddles.

It is clear that every point (x,y) of the plane can be colored according to the result value of iteration Φ_N. Therefore the separatrices of the same color have been intersected.

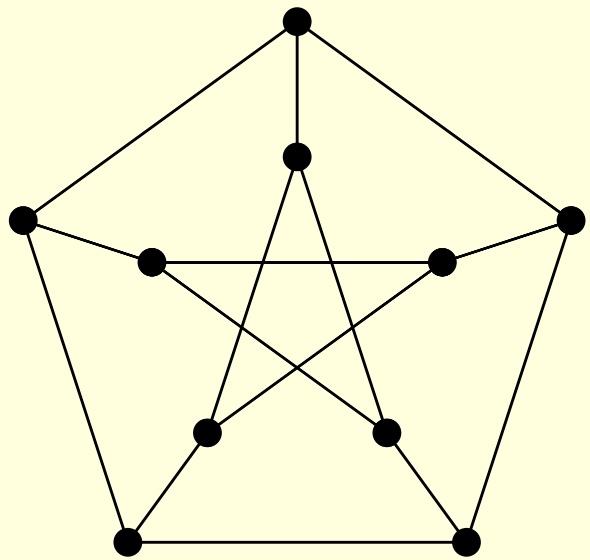
Theorem 3 *Only separatrices from three-separatrix point to three-separatrix point are visible on a painted plane.*

Corollary 1 *The fixed and periodic points are vertexes of trivalent graph on a painted plane.*

Now the most interesting thing!

The problem of coloring for graphs is well-known to be equivalent to the four color problem for arbitrary plane graphs.

The simplest known snark (a snark is a non-edge colorable trivalent graph) is the Petersen graph — shown below:



The Petersen graph is to combinatorics as the Möbius strip is to topology — a ubiquitous phenomenon that insists on turning up when least expected. It is clear that the Peterson graph Euler characteristics is equal -1. Tutte has conjectured that the Petersen graph must appear in any snark.

The coloring plane by the dynamic system action forms trivalent graphs at various parameters B and D.

Question 1 *Is Peterson graph topologically universal or there subsist trivalent graphs with other topology?*

In the other side the Petersen family is a set of seven undirected graphs that includes the Petersen graph and the complete graph K_6.

Question 2 *Is the Petersen family exhaustive for all quasicristals topological types?*